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Public Schemes for Efficiency in Oligopolistic Markets

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I　Introduction

Many governments have been attempting to make public sectors more efficient. Some socialistic economies have moved into market economies while others are struggling to introduce market mechanisms. Capitalistic countries are also trying to make public sectors more competitive. Transferring public sectors to private enterprise can be one solution. However, due to various reasons, this may be difficult or, in some cases, impossible for the state to expedite. Even when private administration of a public sector is easy to implement, the subsequent result is not always efficient. For example, a monopoly or oligopoly may develop.

The unique feature of the incentive scheme that is being proposed in this report lies in restructuring the managerial pay scale. This scheme is not exclusively dependent upon a firm's profit, rather managerial salaries are subject to how a firm's profit compares to its competitor's profit. This discussion uses the following model: a government (or a public institution) owns several pairs of firms in a given industry. The government adopts an incentive scheme where there are only three possible pay scales available to the respective managers of each firm. They are \$$A$, \$$B$, and \$$C$, where $A > B > C$. Of the two firms, the manager whose firm earns the higher profit is paid \$$A$. Whereas, the other manager is paid \$$C$. If both firms earn equal profit, each manager is paid the same \$$B$ salary. With this model we can show that the product price must be equal to the marginal costs in the Nash equilibrium in this game. This assertion holds for arbitrary number of firms (except for only one firm) owned by a government, under another incentive scheme and several moderate assumptions. Furthermore, if there are only two firms, each player's Nash equilibrium strategy weakly dominates all other strategies for each player in many cases. The author showed these results in Takasaki 1995 and 1999 already. This paper is a revised version of them.

There are many researches which investigate incentive schemes towards efficiency in public sectors. However, most of them are concerned about efficiency 'within' an organization or a firm (For example, Lazear and Rosen (1981) analyze an incentive scheme for workers contesting in a competitive firm, Dixit (1997) investigates inefficiency and incentives in an organization (agent) with multiple principals, and Rose-Ackerman (1986) explores incentive schemes in a public bureaucracy). This paper addresses efficiency in an 'oligopolistic industry' which consists of several firms. Furthermore, in many researches, it is assumed that a government knows the cost function of the public firm. On the other hand, this paper assumes that a government can observe only the profits of the state-owned firms.

In the next section, we show a simple example that has commonly used general properties. We prove general propositions in section III. Section IV contains some discussions of our explicit and implicit assumptions, and the implementation of our model in the real world. In section V, we summarize our results and propose some directions for further study.

Ⅱ　Simple Examples

Let us assume there are only two firms in a particular industry. We call them firm 1 and firm 2, and denote the quantity of firm i's product by x_i. The cost function:

$$C = cx_i \quad (i = 1, 2 \quad and \quad c > 0) \tag{1}$$

is common between these firms. The inverse demand function is

$$\left.\begin{array}{ll} p = a - (x_1 + x_2) & if\ x_1 + x_2 \leq a \\ p = 0 & if\ x_1 + x_2 > a \end{array}\right\} \tag{2}$$

where p is the market price of this product and $a > c$. These firms are owned by a unique agent (e.g. a government or a social planner). The owner of each of these firms contracts, with each firm's respective manager, an incentive scheme having the following characteristics:

The Incentive Scheme

Three salary amounts are established so that $A > B > C$. Let π_i denote firm i's profit ($i = 1, 2$). If $\pi_i > \pi_j$ ($j = 1, 2 \quad i \neq j$), the owner pays \$$A$ to the manager of firm i and pays \$$C$ to the manager of firm j. If $\pi_i = \pi_j$, the owner pays \$$B$ to both managers. (The author cautions that C: cost and C: salary be not confused in this discussion). We assume that these salaries are not included in the cost expressed by equation (1) and the owner pays these salaries by subtracting from the firm's profit.

In Figure 1, we recognize that the two diagonal lines correspond respectively to $x_1 = x_2$ (the ray with positive slope) and to $a - c = x_1 + x_2$ (the segment with negative slope).

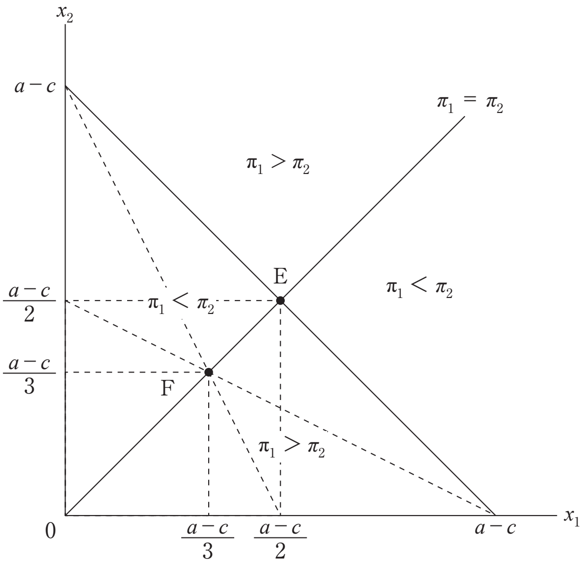

Figure 1

We can check the diagram to confirm that point E indeed corresponds to the unique pure strategy Nash equilibrium of this game. The coordinates of point E are $((a-c)/2, (a-c)/2)$. By substituting this into equation (2), we have the result that $p = c$ (i.e. the price is equal to the marginal cost). Furthermore, note that the strategy for each player, as it corresponds to point E, weakly dominates all other strategies. The ordinary Cournot-Nash equilibrium point F and the reaction curves (the broken lines) are depicted in the figure for comparison.

If there are some private firms with the same cost function as state-owned ones in this market, how does our result change? The answer is 'It does not change'. For example, suppose that a third firm, which is private, exists. Denote its output by x_3. We can easily check that, whatever the value of x_3 is, two state-owned firms adjust their output levels until the market price equals their marginal cost, c. On the other hand, the optimality condition of the private firm is that the marginal revenue is equal to the marginal cost:

$$p'(X)x_3 + p(X) = MC(x_3)$$

where $X \equiv x_1 + x_2 + x_3$. However, $MC(x_3) \equiv c$ in this example and our market mechanism forces the market price, $p(X)$, to equalize to c. Therefore x_3 must be zero. The private firm cannot survive in this industry.

Ⅲ General Result

Our initial assumptions are as follows:

Assumptions

(i) Any function (defined on R_+) in our model is twice continuously differentiable.

(ii) There are n firms where n is an even number. These firms are owned by a particular agent (e.g. a government) in the market for one homogeneous product.

(iii) Let x_i denote a quantity of the output produced by firm i ($1 \le i \le n$) and X the total quantity of the outputs ($X = \sum_{i=1}^{n} x_i$). The market price p is determined when the total supply equals the market demand. The inverse demand function $p = p(X)$ is non-increasing, but strictly decreasing at any equilibrium point in this game.

(iv) Firms s and $s+1$, where s is an odd number with $1 \le s \le n-1$, have the common cost function with the properties: $C_s(x_i) \equiv C_{s+1}(x_i) \ge 0$, $MC_s(x_i) \equiv C'_s(x_i) \ge 0$ and $MC'_s(x_i) \ge 0$ for all $x_i \ge 0$ where $i = s$ or $s+1$. This cost does not include the managers' salaries (i.e. we do not define these salaries as a part of cost).

We denote $(x_1, x_2, \ldots\ldots, x_n)$ by x. Firm i's profit, $p(X)x_i - C_i(x_i)$, is denoted by $\pi_i(x)$, or π_i simply, for all i with $1 \le i \le n$. We often refer the manager of firm i as player i.

Incentive Scheme Ⅰ

The payoff function of player s and player $s+1$, where s is an odd number with $1 \le s \le n-1$, are as follows;

$$\left.\begin{array}{llll} \Pi_s(x) = A & and \quad \Pi_{s+1}(x) = C & if & \pi_s > \pi_{s+1} \\ \Pi_s(x) = B & and \quad \Pi_{s+1}(x) = B & if & \pi_s = \pi_{s+1} \\ \Pi_s(x) = C & and \quad \Pi_{s+1}(x) = A & if & \pi_s < \pi_{s+1} \end{array}\right\} \tag{3}$$

Where $A > B > C$.

We consider only pure strategies under the incentive scheme described by equations (3). We have the following propositions followed by their proofs.

Proposition 1

Let $x^* = (x_1^*, x_2^*, \ldots\ldots, x_n^*)$ denote a Nash equilibrium pure strategy profile for this game with the incentive scheme operating under the previous assumptions (i) to (iv). Then x^* satisfies the following equations: $\pi_s(x^*) = \pi_{s+1}(x^*)$ where s is an odd number with $1 \le s \le n-1$.

Proof

Suppose that $\pi_s(x^*) < \pi_{s+1}(x^*)$ for some s, then player s (the manager of firm s) earns \$$C$. Yet, player s can duplicate player $s+1$'s strategy x^*_{s+1} (i.e. $x_s = x^*_{s+1}$). By this strategy change player s can achieve \$$B$ because $\pi_s = \pi_{s+1}$ when $x_s = x_{s+1}$. When $\pi_s(x^*) > \pi_{s+1}(x^*)$, we obtain a corresponding contradiction by using the same argument. Q.E.D.

Proposition 2

If $x_i^* > 0$ for all i ($1 \le i \le n$), then we obtain the following equations:

$$p(X^*) = MC_1(x_1^*) = MC_2(x_2^*) = \;.\,.\,.\,.\,.\,.\,. = MC_n(x_n^*)$$

where $X^* = \sum_{i=1}^{n} x_i^*$. (With a realistic assumption that the government can observe the fact that $x_i^* = 0$ for any i, the condition that $x_i^* > 0$ for all i can be satisfied by introducing an even lower salary $E < C$ which could be negative and any player must get whenever his/her firm's output level is zero.)

Proof

Let us define ${}_i\Delta_j = \pi_i(x) - \pi_j(x)$. The partial derivatives of ${}_s\Delta_{s+1}$ by x_s and ${}_{s+1}\Delta_s$ by x_{s+1} are:

$$\begin{aligned} \frac{\partial_s \Delta_{s+1}}{\partial x_s} &= p'(X)(x_s - x_{s+1}) + p(X) - MC_s(x_s) \\ \frac{\partial_{s+1} \Delta_s}{\partial x_{s+1}} &= p'(X)(x_{s+1} - x_s) + p(X) - MC_{s+1}(x_{s+1}) \end{aligned} \tag{4}$$

We will show that at equilibrium strategy profile x^* these two equations are equal to zero. By the previous proposition 1, we know ${}_s\Delta_{s+1} = 0$ (i.e. $\pi_s = \pi_{s+1}$). If $\partial_s \Delta_{s+1} / \partial x_s > 0$ (or <0, respectively), then player s can achieve a situation where ${}_s\Delta_{s+1} > 0$ by increasing (or decreasing, respectively) firm s' s output by an arbitrarily small amount. By doing so player s can increase his/her payoff from \$$B$ to \$$A$. Therefore, the first equation in (4) must be equal to zero at x^*. The same reasoning holds for the second equation.

Now we have the following equation:

$$2p'(X^*)(x_s^* - x_{s+1}^*) = MC_s(x_s^*) - MC_{s+1}(x_{s+1}^*) \tag{5}$$

If $x_s^* > x_{s+1}^*$, by our assumptions that $p'(X^*) < 0$ and $MC_s'(x_s) \ge 0$, it must be that $0 > 2p'(X^*)(x_s^* - x_{s+1}^*)$ $= MC_s(x_s^*) - MC_s(x_{s+1}^*) \ge 0$. This is a contradiction. In the case that $x_s^* < x_{s+1}^*$, we also obtain a contradiction by a similar argument. Therefore, it must be that $x_s^* = x_{s+1}^*$. By substituting this equality into the equations of (4) we obtain the fact that $p(X^*) = MC_s(x_s^*) = MC_{s+1}(x_{s+1}^*)$ since the equations of (4) are equal to zero at x^*. This result holds for any odd number s with $1 \le s \le n - 1$. Q.E.D.

The propositions are still valid if we replace the managerial pay scale described by equation (3) with a more general pay scale defined by:

$$\Pi_s(x) \equiv f({}_s\Delta_{s+1}) \text{ and } \Pi_{s+1}(x) \equiv f({}_{s+1}\Delta_s)$$

where f is a function such that $f(a) > f(0)$ if $a > 0$, and $f(a) < f(0)$ if $a < 0$.

We can easily extend these propositions to a multiproduct case, assuming that the demands are independent of each other (i.e. no gross substitute and no gross complementarity). The proof is almost identical as shown above. However, when the goods are related in consumption, it seems difficult to obtain the same result (i.e. the price is equal to the marginal cost) as in our previous model. The author shall leave the further study of a general multiproduct case to the future.

Another important point is that this incentive scheme must be immune to collusion between managers. In a case where $A + C > 2B$, for example, each manager may try to realize a situation wherein $\pi_s > \pi_{s+1}$ or $\pi_s < \pi_{s+1}$, and both manager agree to share their respective salaries by an (implicit) contract using side-payment or transfers. When $A + C \leq 2B$, there is no incentive for this kind of collusion; this satisfies the incentive compatibility condition. Furthermore, B must not be less than the most favorable remuneration obtainable in other industries (denoted by D), i.e. $B \geq D$. This satisfies the individual rationality condition.

The financial balance of the owner (e.g. government) must be taken into account. If, in an equilibrium, each firm earns a larger profit than D, then a government can avoid a deficit by offering B; thus satisfying the following inequality:

$$\pi_i(x^*) \geq B \geq D$$

This relationship corresponds to a self-supporting accounting system in a particular public industry. If the above condition is not satisfied, a government must recover its deficit by taxation. Our example in the previous section can satisfy this condition only when $D = B = 0 > C$. The financial balance of the owner of each firm depends mainly on the shape of the cost function and the demand function; we will refer to this matter in section IV.

Next, we allow a government to own odd number of firms except that it owns only one firm.

Assumption (ii)′ A government owns n firms where $n > 2$ and n is an odd number.

We also replace assumption (iv) with the following assumption.

Assumption (iv)′ All firms have the identical cost function denoted by $C(x_i)$.

This assumption appears more restrictive than assumption (iv), but we do not assume that the marginal cost is non-decreasing.

Furthermore, we consider another assumption that a 'monopoly', if any, can be profitable in this industry.

Assumption (v) $\exists X: p(X)X - C(X) > 0$

Let us consider the following incentive scheme.

Incentive Scheme Ⅱ

When there are n firms with $n \geq 3$, payoff functions for players are:

$$\left.\begin{array}{lll}\Pi_i(x) = A & if & \pi_i(x) > \pi_{i+1}(x) \\ \Pi_i(x) = B & if & \pi_i(x) = \pi_{i+1}(x) \\ \Pi_i(x) = C & if & \pi_i(x) < \pi_{i+1}(x)\end{array}\right\} \quad (6)$$

where $A > B > C$ and $1 \leq i \leq n$. But when $i = n$, $i + 1$ should be interpreted as 1.

Under Incentive Scheme II, we have the following proposition.

Proposition 3

At any Nash equilibrium with pure strategies, it must be that $\Pi_i(x^*) = B$ for all i.

Proof

If $\pi_i > \pi_{i+1}$ for some i, it must be that $\pi_k < \pi_{k+1}$ for some $k < n$ or $\pi_n < \pi_1$. Otherwise, it must be that $\pi_1 \geq \pi_2 \geq \ldots\ldots \pi_i > \pi_{i+1} \geq \pi_{i+2} \geq \ldots\ldots \geq \pi_n \geq \pi_1$. This is a contradiction. Then player k can duplicate the strategy of player $k + 1$ (or player 1 when $k = n$) to increase his/her payoff. If $\pi_i < \pi_{i+1}$ for some i, player i can duplicate player $i + 1$'s strategy. Q.E.D.

Under assumptions (i), (ii)′, (iii), (iv)′, (v), and Incentive Scheme II, we have the following proposition.

Proposition 4

At any symmetric Nash equilibrium with pure strategies (i.e. $x_1^* = x_2^*, = \ldots\ldots = x_n^*$), the marginal costs of all firms must be equal to the market price of the product.

Proof

Let x^* be a symmetric equilibrium. Firstly, note that $x_i^* > 0$ for all i. If $x_i^* = 0$ for all i, then $\pi_i(0) \leq 0$ for all i. By assumption (v), any player has an opportunity to increase his/her payoff choosing some positive output.

Since ${}_i\Delta_{i+1} = 0$ for all i, by the same argument as in the proof of proposition 1, we obtain that $\partial\, {}_i\Delta_{i+1} / \partial x_i = p(X^*) - MC(x_i^*) = 0$ for all i (when $i = n$, $i + 1$ means 1). Q.E.D.

Under assumptions (i), (ii)′, (iii), (iv)′, and Incentive Scheme II, we have the following proposition.

Proposition 5

Let x^* be an arbitrary Nash equilibrium pure-strategy profile. If $x_i^* > 0$ and the marginal cost is constant i.e. $MC(x_i) \equiv c$ for all i, then it must be that $p(X^*) = c$.

Proof

Now we know that $\partial_i \Delta_{i+1} / \partial x_i = p'(X^*)(x_i^* - x_{i+1}^*) + p(X^*) - c = 0$ for all i (when $i = n$, $i+1$ should be interpreted as 1). Adding these equations for all i, we obtain the result $p(X^*) = c$. Q.E.D.

Even if some private firms exist in this market, these results still hold for state-owned firms assuming that cost functions are identical among all firms and the marginal cost functions are non-decreasing. Whatever private firms' output levels are, state-owned firms equalize their marginal costs to the market price in this game. On the other hand, private firms equalize their marginal costs to their marginal revenues: $p(X^*) - MC(x_i^*) = -p'(X^*)x_i^*$. Since $p'(X^*) < 0$ and the marginal cost function is non-decreasing, the above equation means that the output levels of private firms are less than those of state-owned firms'. Each private firm's profit is also less than each state-owned firm's because the market price, $p(X^*)$, is common to both types of firms. Therefore private firms can not dominate the state-owned firms in the long run.

Ⅳ Considerations on the Assumptions

Now we have the important result that the price must be equal to the marginal costs at equilibrium in our model. However, for an application of our model to a real industry, we should examine the assumptions we have postulated. (Assumption (i) is necessary for only technical reasons; we will not put a comment on it in this paper. In fact, even though our examples in section II and the appendix violate this assumption, we obtain the same result.)

In assumption (ii) the owner of both firms could be a federal government, local government, or any welfare-motivated public institution. The profit of each firm must be both observable and verifiable by the owner in order to implement the incentive scheme. In the real world, it is difficult to detect whether a firm's accounting reports have been manipulated intentionally. The owner will encounter the 'principal-agent problem', as is often the case in this kind of model. Therefore, the owner must provide a complete audit system for each firm.

Assumption (iii) consists of two parts. One addresses a particular price mechanism, and the other the property of a demand function. The former is clearly a Cournot Price Mechanism. Whether or not we can apply the Cournot model to a particular industry depends on the circumstances of that industry. We must investigate the characteristics of the industry before applying our incentive scheme. In a case where the market is a Bertrand type and there is no capacity constraint for each firm with constant average cost, we do not need an incentive scheme because the Bertrand equilibrium equalizes the price and the marginal cost. However, if the price mechanism can be interpreted as a Cournot type, our incentive scheme dominates the corresponding Cournot-Nash equilibrium (see Figure 1 and compare point F with E). Even if the price mechanism does not work as a Cournot model, a government may be able to play the role of auctioneer for the Cournot market. Thus, the price mechanism aspect of assumption (iii) is comparatively non-restrictive.

Assumption (iii) also assumes that the slope of the demand curve is negative in a neighborhood of an equilibrium point. Note, however, that we do not need this part of assumption (iii) in some cases. In fact, the example in the appendix does not satisfy this and still produces efficient outcomes.

Assumption (iv) also consists of two parts. One restricts the shape of the cost function and the other requires that a pair of firms have the same technology. The restrictions on the cost function seem to be natural and innocuous. Yet, should there be a sunk cost (as is often the case with the real world), we cannot always guarantee a non-negative profit for each firm in an equilibrium. This is related to a deficit case, which we have already considered. For example, consider the simple cost function as follows:

$$C(x_i) = cx_i + F \quad c > 0 \quad F > 0$$

Given this cost function, each firm must earn a negative profit in the Nash equilibria because it must be that $p = c$. In this case the marginal cost is always lower than the average cost. This corresponds to a natural monopoly because the cost function is subadditive. It follows that the 'cost minimizing-number of firms' is one. (When we assume that all firms have the same cost function, the cost-minimizing number of firms for an industry to produce X units of its product is defined as an element of the following set which can be empty:

$$N(X) \equiv \left\{ n : IC(X) = \sum_{i=1}^{n} C(x_i),\ \sum_{i=1}^{n} x_i = X \right\}$$

where

$$IC(X) \equiv min \left\{ \sum_{i=1}^{n} C(x_i) : \sum_{i=1}^{n} x_i = X,\ n \text{ is a natural number} \right\}.$$

$IC(X)$ is called 'minimum industry cost'. See Baumol and Fisher (1978).) The cost minimizing number of firms varies with both the shapes and locations of the demand and cost functions. This is the reason why we need analyses for the case a government owns more than two firms.

It is necessary for our proof that each pair of firms (or even all firms) have an identical cost function. The owner of the firms must supply to at least two firms not only equal funding but also impartial market information and production technology. These firms must have equal opportunity. A government may actualize this by exchanging managers and engineers among firms periodically. However, this may decrease the incentive to innovative efforts. In order to avoid this, a special bonus (for example for successful innovations) must be introduced to the incentive scheme. The author, however, shall leave further study of dynamic cases to the future.

V Concluding Remark

We have proposed a special but very simple incentive schemes for the public sector. We have also demonstrated that the price must be equal to the marginal cost in the equilibrium of this game. The crucial assumptions in implementing these incentive schemes are:

1) A government (or a social planner) must own more than one firm.
2) The firms competing with each other must have identical access to production technology and market information.

Assumption 2) may be proven to be too confining in subsequent investigations. The author hopes that further studies will solve this problem. Future investigations in the following area will advance the development of, and generalize, this incentive scheme mechanism. Suggested topics are: a) general multiproduct cases, and b) firms with different cost functions.

Appendix

We cannot always guarantee the uniqueness of the pure strategy Nash equilibrium. Let us consider the case where the marginal cost c=0 and other conditions are the same as in our example in section Ⅱ. In Figure 2, any point on the shaded region including its boundary is a Nash equilibrium. Note that in Figure 2 there are two triangles ΔOEa which share one common segment OE. In the upper-left triangle and the lower-right triangle denotes $\pi_1 < \pi_2$ and $\pi_1 > \pi_2$ respectively, with the exception that, in each case, side OE and aE are not included. When in the upper-left triangle region, player 1 earns payoff \$$C$ and player 2 earns payoff \$$A$. These payoffs are reversed when in the lower-right triangle region. Segment OE means $\pi_1 = \pi_2$. We can easily confirm that any point in the shaded area in Figure 2 is a Nash equilibrium.

Fortunately, at any point in this shaded area, the price is equal to the marginal cost, which is zero in this particular example. Therefore, the multiplicity of pure strategy Nash equilibria poses no problem to our result in this model. Especially, each player's strategy, corresponding to point E, weakly dominates all other strategies. Point E corresponds to the unique pair of weakly dominant strategies.

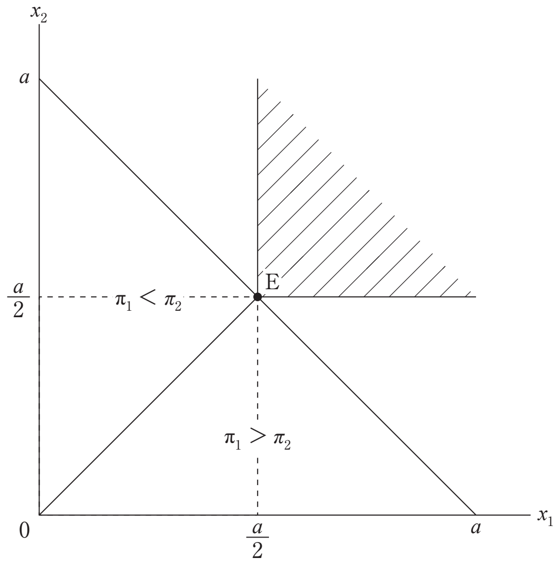

Figure 2

References

[1] Baumol, W.J., and Fisher, D., 1978 'Cost-Minimizing Number of Firms and Determination of Industry Structure', Quarterly Journal of Economics, 92

[2] Dixit, A., 1997 'Power of Incentive in Private versus Public Organizations', AEA Papers and Proceedings 87(2)

[3] Lazear, A.P., and Rosen, S., 1981, 'Rank-Order Tournaments as Optimum Labor Contracts', Journal of Political Economy. 89

[4] Rose-Ackerman, S., 1986, Reforming Public Bureaucracy through Economic Incentives?, Journal of Law, Economics and Organization 2(1).

[5] Takasaki, J., 'An Incentive Scheme towards Efficiency in Public Industry', 1995, Working Paper.

[6] Takasaki, J., 'A Public Scheme towards Efficiency in Oligopolistic Markets', 1999, Working Paper.