An Empirical Analysis of Bundling Sales in Online Auction Markets

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1　Introduction

Today, many people use consumer-to-consumer electronic commerce sites to buy（or sell）goods. In particular, with the emergence of online auction sites（e.g., eBay and Yahoo!）, many people have become familiar with auctions. In such tradings, sellers often sell two or more items as bundling auctions. However, other sellers sell the same items separately. In this paper, we focus on bundling auctions of online auction markets. We propose an empirical model of online common value auctions for both bundling auctions and separate auctions.

Some papers focus on the bundling auction model in theoretical literature. Palfrey（1983）studied bundling auctions with two bidders. He found that bundling auctions generate more expected revenues with two bidders within the private values paradigm. Chakraborty（1999）extended Palfrey（1983）to a general number of bidders. He found that if the number of bidders grows large, the expected revenue of separate sales becomes greater than that of bundling auctions. While Chakraborty（1999）studied the private values model, Chakraborty（2002）studied the common value auction model and found the effect that they call *the winner's curse reduction effect* in bundling auctions. He also compared the expected revenues between bundling auctions and separate auctions.

Our empirical example involves eBay mint coin auctions in 2014. In our data set, there are two kinds of coin sets: 11-coin sets and 22-coin sets. We regard the 11-coin sets as separate items and the 22-coin sets as bundled items. We also conduct some counterfactual simulations using the estimated parameters. We evaluated the winner's curse reduction effect in the sense of Chakraborty（2002）and compared revenue between bundling auctions and separate auctions. Chakraborty（2002）

showed that bidders will bid more aggressively in separate auctions than in bundling auctions; he named this effect *the winner's curse reduction effect*. We measured the magnitude of the winner's curse reduction effect. We found that bidders in separate auctions will bid $2.5 higher than in bundling auctions. For revenue comparison, we found that the expected revenue in a bundling auction is higher than that in separate auctions by $0.37. Since the average transaction price of bundled items（22-coin sets）is $8.98, the value of additional gains are not negligible.

The rest of this paper is organized as follows. In Section 2, we describe the model of online auctions within the pure common value paradigm. Additionally, following Chakraborty（2002）, we review the theoretical results for the bundling auctions. Section 3 describes the estimation strategy for the model described in Section 2. We utilize Bayesian estimation to estimate the structural parameters. In Section 4, we estimate the structural parameters using real auction data. Our empirical example is eBay mint coin set auctions in 2014. In Section 5, we compute the winner's curse reduction effect in the sense of Chakraborty（2002）and compare the revenue between separate auctions and bundling auctions using the estimated parameters. Section 6 features some concluding remarks.

2 Model

There are N risk neutral potential bidders and a seller. The number of potential bidders, N, is a random variable and an exogenous variable. The seller sells two different objects $k = 1$ and 2. In this model, we consider the pure common value model in which the *ex post* valuation of the item is the same for each bidder. Let V_1 and V_2 denote the values for items 1 and 2, respectively. The realizations of values are unknown to the bidders. Instead, each bidder, i, receives her private signals corresponding to V_1 and V_2, which are denoted by S_{1i} and S_{2i}, respectively. Each bidder knows the realization of her own private signal but does not know the others' before auctions. However, both the distribution of S_{1i} and the distribution of S_{2i} are common knowledge among bidders. In this paper, we consider a specific functional form for V_1 and V_2. We assume that the value of each item to bidders is the average of their signals. That is, the valuations take the form of

$$V_1 = \frac{1}{N} \sum_{i=1}^{N} S_{1i} \text{ and } V_2 = \frac{1}{N} \sum_{i=1}^{N} S_{2i},$$

respectively.[1] For the bundled item, we impose the additive separability on bidder i's signal for the bundled items, S_i. Namely, we assume that $S_i = S_{1i} + S_{2i}$. Then, from our specific functional form for the value, the valuation of the bundled item, V, is

$$V = \frac{1}{N} \sum_{i=1}^{N} S_i = \frac{1}{N} \sum_{i=1}^{N} (S_{1i} + S_{2i}) = V_1 + V_2$$

We assume that $S_{k1}, ..., S_{kN}$ are independently and identically distributed. Namely, $S_{ki} \sim i.i.d.$

$F_k(x)$ for $k \in \{1, 2\}$. We assume that S_{1i} and S_{2i} are independently distributed. Furthermore, we assume that for each $k \in \{1, 2\}$, S_{ki} is affiliated with $S_{1i} + S_{2i}$ in the sense of Milgrom and Weber (1982).

2.1 Equilibrium

In this paper, we regard online eBay auctions as second-price auctions. That is, each bidder submits her bid and the bidder with the highest bid among bidders wins the object and pays the second highest bid. Then, the equilibrium bidding strategies in separate auctions for items $k = 1$ and 2 are straightforward arguments from Milgrom and Weber (1982).

Let Y_{ki} be the highest signal except bidder i's signal, S_{ki}. That is, $Y_{ki} = \max_{j \neq i} S_{kj}$. Then, the equilibrium bidding functions for bidder i with private signals $S_{1i} = s_1$ and $S_{2i} = s_2$ in separate auctions are given by

$$b_1(s_1) = \mathrm{E}[V_1 | S_{1i} = s_1, Y_{1i} = s_1] \text{ and } b_2(s_2) = \mathrm{E}[V_2 | S_{2i} = s_2, Y_{2i} = s_2] \tag{1}$$

for items $k = 1$ and 2, respectively.

Analogously, the equilibrium bidding function for bundling auctions can be derived in the same manner. Let $S_i = S_{1i} + S_{2i}$ be the sum of bidder i's signals, S_{1i} and S_{2i}. Furthermore, let $G(\cdot)$ denote the cumulative distribution function of $S_i = S_{1i} + S_{2i}$. In other words, $G(\cdot)$ is the convolution of $F_1(\cdot)$ and $F_2(\cdot)$. Then, $S_1, ..., S_N$ are also independently and identically distributed with the CDF $G(\cdot)$. Namely, $S_i \sim i.i.d. \ G(s)$.

Let Y_i be the highest signal except bidder i's signal, S_i. That is, $Y_i = \max_{j \neq i} S_j$. Then, using an argument similar to that of a separate auction, we gain the equilibrium bidding function for bidder i with signal $S_i = s$ in the bundling auctions

$$b(s) = \mathrm{E}[V_1 + V_2 | S_i = s, Y_i = s]. \tag{2}$$

2.2 Bundling auctions versus separate auction

Since we computed the various effects of bundling auctions in our empirical example, it is worth-while to review the theoretical result of bundling auctions within the common value paradigm. Chakraborty (2002) discussed the bundling auctions model and the separate auctions model within the pure common value paradigm. Furthermore, he discussed the effect of bundling auctions and separate auctions with some useful examples. In this subsection, we review the results of Chakraborty (2002).

Chakraborty (2002) discussed that bundling auctions have a winner's curse reducing effect. The intuitive explanation of the winner's curse reducing effect is as follows. In separate auctions of $k = 1$ and 2, winning the items $k = 1, 2$ implies that each winner has the highest signal on each item. On the other hand, in a bundling auction, winning the bundled item implies that the winner has the highest signal for the bundled item but not for individual items, $k = 1$ and 2. Therefore, winning the

bundling auction is not as bad as winning two separate auctions. The following theorem is the Theorem 1 in Chakraborty (2002). They call the result of Theorem 1 *the winner's curse reducing effect.*

Theorem 1 (Chakraborty (2002)). *A bidder bids more aggressively when the objects are bundled. That is,*

$$b(s) \geq b_1(s_1) + b_2(s_2),$$

where $s = s_1 + s_2$.

3 Estimation

The results of equilibrium bidding strategies (1) and (2) are familiar to economists. However, few empirical studies focus on the structural estimation of common value auction models. The main reason is the negative result of nonparametric identification on the common value auction model. Athey and Haile (2002) and Athey and Haile (2007) showed the conditional distribution of S_{ki}, given V_k is not identified from the observed bids in the common value auction model without additional identification conditions.

Therefore, most studies of structural estimation of the auction model focus on the private values model. Recently, some papers have studied the identification condition of the common value auction model. For example, Li, Perrigne, and Vuong (2000) showed the identification under the additive separability of the common value component. Février (2008) restricted the shape of the density function of the common value and showed the identification of the common value auction model. d'Haultfoeuille and Février (2008) proposed the identification condition of the common value auction model, assuming the support of a private signal is finite and varies depending on the common value. In this paper, we impose parametric specification to avoid the identification problem.

3.1 Estimation procedure

We observe T_k auctions indexed by $t = 1, ..., T_k$ for item $k \in \{1, 2\}$. The same items are each sold in separate auctions. Analogously, we observe T auctions indexed by $t = 1, ..., T$ for bundling auctions. We can observe each bidder's bid, B_{kit}, and the number of actual bidders, n_t, for bidder i, for item $k \in \{1, 2\}$, and for auction $t \in \{1, ..., T_k\}$. We cannot observe each bidder's signals, S_{kit} and S_{it}, the common value, V_{kt} and V_t, and the number of potential bidders, N_t. We assume that the number of potential bidders is constant among auctions as is the maximum number of actual number of bidders observable by econometricians such as Guerre, Perrigne, and Vuong (2000).

Following the example of Bajari and Hortaçsu (2003), we assume that bidders' signals, S_{kit}, are normally distributed with mean, μ_{kt}, and variance, σ_{kt}^2. That is, for $k \in \{1, 2\}$, $S_{kit} \sim N(\mu_{kt}, \sigma_{kt}^2)$, where

$\mu_{kt} = \boldsymbol{\alpha}_k' X_{kt}$ and $\sigma^2_{kt_k} = (\exp(\beta_{k1}), ..., \exp(\beta_{kd}))X_{kt}$, where d represents the dimensionality of the vector of the coefficient parameter, and X_{kt} is the vector of the auctionspecific covariate. The values of $\boldsymbol{\alpha}_k = (\alpha_{k1}, ..., \alpha_{kd})$ and $\boldsymbol{\beta}_k = (\beta_{k1}, ..., \beta_{kd})$ are unknown to econometricians; therefore, we estimate these parameters.

Recall that the equilibrium bidding function $b_k(\cdot)$ is given by

$$b_k(s_k) = \mathrm{E}(V_{kt}|S_{kit}=s_k, Y_{kit}=s_k).$$

Since $b_k(\cdot)$ is a strictly increasing function, there exists an inverse function $\phi_k(\cdot)$. Note that since we considered second-price auctions, the winning bid of item k, w_{kt}, in auction t is the second-highest bid in auction t. Therefore, observing the winning bids, the likelihood function for separate auctions is given by

$$L(w_1, ..., w_{kt}|\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k, (X_{k1}, ..., X_{kt})) = \prod_{t=1}^{T_k} \binom{N}{1 \quad 1 \quad N-2} [F_k(\phi_k(w_{kt})|\mu_{kt}, \sigma^2_{kt})]^{N-2}$$
$$\times f_k(\phi_k(w_{kt})|\mu_{kt}, \sigma^2_{kt}) \frac{1}{b_k'(\phi_k(w_{kt}))}$$
$$\times [1 - F_k(\phi_k(w_{kt})|\mu_{kt}, \sigma^2_{kt})], \tag{3}$$

where $f_k(\cdot)$ is the probability density function of S_{kit}. In this case, $f_k(\cdot)$ is the normal density function.

The likelihood function for bundling auctions can be derived in the same manner. We assume that S_{it} is the normal random draw with mean μ_t and variance σ^2_t. That is, $S_{it} \sim N(\mu_t, \sigma^2_t)$ where $\mu_t = \boldsymbol{\alpha}'X_t$ and $\sigma^2_t = (\exp(\beta_1), ..., \exp(\beta_d))X_t$, where $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_d)$ and $\boldsymbol{\beta} = (\beta_1, ..., \beta_d)$ are the unknown coefficient parameter vector to be estimated.

Since the equilibrium bidding function, $b(\cdot)$, is a strictly increasing function, there exists an inverse function, $\phi(\cdot)$. Similar to the separate auctions, since we considered second-price auctions, the winning bid, w_t, in auction t is the second-highest bid in auction t. Therefore, in observing the winning bids, the likelihood function for the bundling auction is given by

$$L(w_1, ..., w_t|\boldsymbol{\alpha}, \boldsymbol{\beta}, (X_1, ..., X_t)) = \prod_{t=1}^{T} \binom{N}{1 \quad 1 \quad N-2} [G(\phi(w_t)|\mu_t, \sigma^2_t)]^{N-2}$$
$$\times g(\phi(w_t)|\mu_t, \sigma^2_t) \frac{1}{b'(\phi(w_t))}$$
$$\times [1 - G(\phi(w_t)|\mu_{kt}, \sigma^2_{kt})], \tag{4}$$

where $g(\cdot)$ is the probability density function of S_{it}. In this case, $g(\cdot)$ is the normal density function.

4　Empirical examples

4.1　Data description

Our empirical example consists of auctions of 2005 U.S. mint coin set held on eBay in 2014. Data were collected from 208 eBay auctions completed in October, 2014. There are two types of goods in our data set. One is the 11-coin mint set and the other is the 22-coin mint set. The 22-coin mint set includes two packages of the 11-coin mint set. The sample sizes are 107 and 101, respectively.

As Bajari and Hortaçsu（2003）and Wegmann and Villani（2011）studied coin auctions in their empirical illustrations, coin auctions are excellent examples in the empirical study of the common values auction model. While Bajari and Hortaçsu（2003）and Wegmann and Villani（2011）both collected various kinds of coins in their empirical illustrations, we only collected 2005 U.S. mint coin sets （11-coin sets and 22-coin sets）. Therefore, we estimated the distribution of signals with fewer covariates.

Tables 1 and 2 provide the summary of statistics for the 11-coin set and 22-coin set, respectively. The first column describes the variables. "Winning bid" is the second highest bid in the eBay auction. As seen in Tables 1 and 2, on average, one could purchase a mint coin set for $7.3 or $9.2 for the 11-coin set or 22-coin set, respectively. "Positive reputation" denotes the number of positive ratings a seller has received. Similarly, "Negative reputation" is the sum of the number of negative ratings and the number of neutral ratings a seller receives. Since the number of neutral ratings and the number of negative ratings are usually small relative to the number of positive ratings, we

Table 1: Summary statistics（2005 U.S. mint coin sets,（11-coin set）# of obs. = 107）

	Mean	Std	Median	Max	Min
Winning bid	6.55	2.64	5.99	15.5	2.25
Positive reputation	6,515.74	17,105.87	388.00	73913	5
Negative reputation	9.52	28.49	0.00	194	0.00
Number of actual bidders	2.87	1.61	3.00	6.00	1.00
Days	5.26	2.28	7.00	10.00	0.00

Table 2: Summary statistics（2005 U.S. mint coin sets,（22-coin set）# of obs. = 101）

	Mean	Std	Median	Max	Min
Winning bid	8.98	3.35	8.25	17.0	3.3
Positive reputation	22,553.29	33,006.16	1,303.00	73,892.00	0.00
Negative reputation	13.39	17.17	3.00	57.00	0.00
Number of actual bidders	3.47	2.04	3.00	7.00	1.00
Days	5.98	2.06	7.00	10.00	1.00

regard neutral ratings as negative ratings. "Number of actual bidders" is the number of participants who actually bid at auction t. "Days" denotes the duration of the auctions held.

4.2 Estimation results

4.2.1 The 11-coin set

For the 11-coin set, we assume that the signal, S_i, follows the normal distribution. That is,

$$S_{it} \sim i.i.d. \ N(\mu_{1t}, \sigma_{1t}^2),$$

where $\mu_{1t} = \alpha_0 + \alpha_1 X_{t1} + \alpha_2 X_{t2}$, and $\sigma_{1t}^2 = \exp(\beta_0) + \exp(\beta_1) X_{t1} + \exp(\beta_2) X_{t2}$. The parameters $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2)$ are unknown to econometricians.[2] In this empirical illus- tration, the auction-specific covariates, $X_t = (X_{t1}, X_{t2})$ are the logarithm of "Positive reputation+1" and "Negative reputation + 1"; that is,

$$X_{t1} = \log(\text{Positive reputation}+1) \text{ and } X_{t2} = \log(\text{Negative reputation}+1)$$

for observed auction t.

The prior distribution of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $\boldsymbol{\alpha} \sim N(0, 10\boldsymbol{I})$ and $\boldsymbol{\beta} \sim N(0, 10\boldsymbol{I})$, where $\boldsymbol{I}$ is the identity matrix of order 3.

We used the random walk-based Metropolis-Hastings algorithm to generate random draws from the posterior distributions. The number of iteration is 20000, and the burn-in period is 1000. Table 3 reports the probabilities parameters take positive (PP), the *p*-values of convergence diagnostics for the MCMC (CD) and Inefficiency Factors (IF). All *p*-values of the convergence diagnostics are more than 0.06. Furthermore, the inefficiency factor values are sufficiently low. In particular, the inefficiency factors are 39.88 to 95.65, which implies that we would obtain the same variance of the posterior sample means from 209 uncorrelated draws, even in the worst case. Figure 1 shows the sample paths of estimated parameters. From Figure 1 it can be seen that the sample paths of these parameters converge to posterior distributions. Thus, we conclude that the sample paths of estimated parameters converge to posterior distributions.

Figure 2 shows the posterior densities of parameters for the 11-coin set. Table 4 and Figure 2

Table 3: The convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF) for the 11-coin set

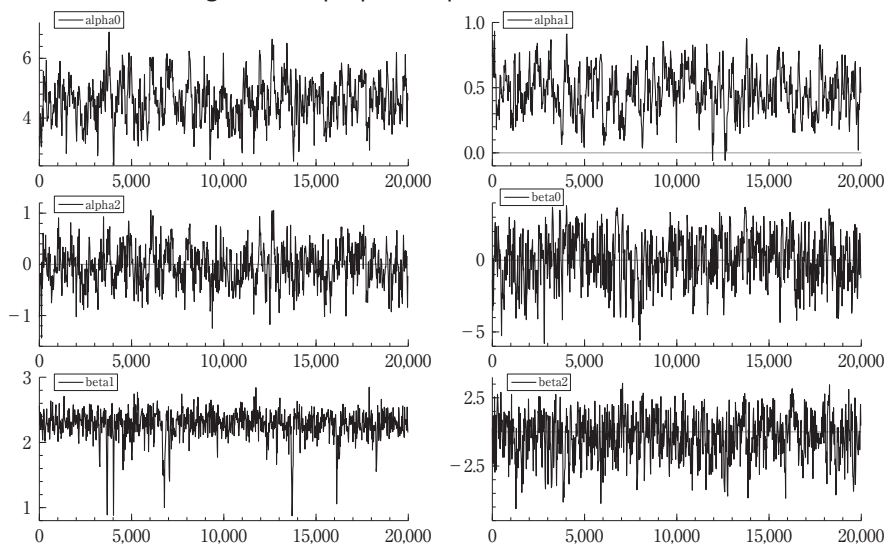| Parameter	Covariate (Coefficient Parameter)	PP	CD	IF
μ_1	Const. (α_0)	1.00	0.77	87.41
	$\log(\text{Pos.Rep.} +1)$ (α_1)	1.00	0.85	95.65
	$\log(\text{Neg.Rep.} +1)$ (α_2)	0.46	0.96	66.27
σ_1^2	Const. (β_0)	0.51	0.06	54.85
	$\log(\text{Pos.Rep.} +1)$ (β_1)	1.00	0.32	47.15
	$\log(\text{Neg.Rep.} +1)$ (β_2)	0.44	0.77	39.88

Figure 1: Sample paths of parameters（11-coin set）

Table 4: Posterior inferences for the 11-coin set

Parameter	Covariate（Coefficient Parameter）	Mean	Stdev.	95% credible interval
μ_1	Const.（α_0）	4.59	0.71	（3.23, 6.00）
	$\log(\text{Pos.Rep.}+1)$（α_1）	0.46	0.17	（0.12, 0.79）
	$\log(\text{Neg.Rep.}+1)$（α_2）	-0.03	0.39	（-0.78, 0.73）
σ_1^2	Const.（β_0）	0.01	1.76	（-3.56, 3.12）
	$\log(\text{Pos.Rep.}+1)$（β_1）	2.27	0.22	（1.71, 2.60）
	$\log(\text{Neg.Rep.}+1)$（β_2）	-0.35	1.60	（-3.70, 2.44）

Figure 2: Posterior densities（11-coin set）
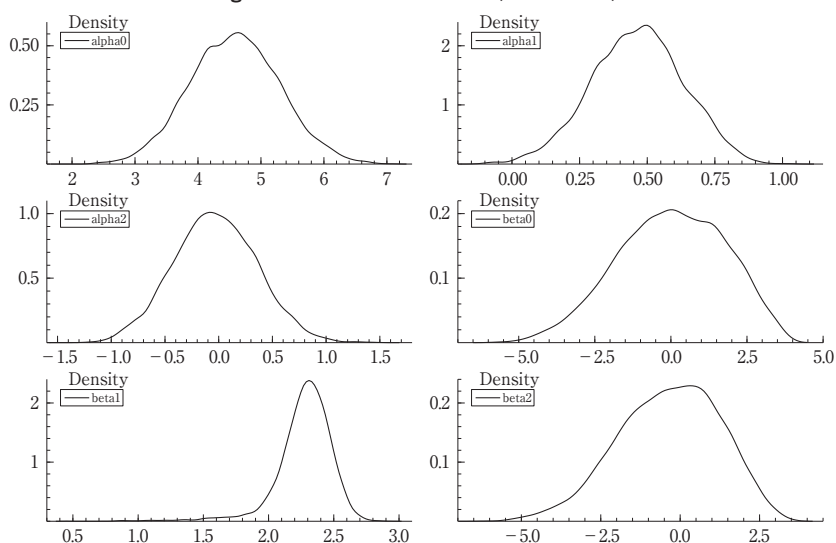
provide some posterior inferences. In Table 4, "Mean," "Stdev," and "95% interval" represent the posterior mean, the posterior standard deviation, the 95% credible interval, respectively.

As seen in Table 4, the posterior mean of α_0 is 4.59. Since α_0 is the constant term corresponding to the mean parameter μ_1, when a seller has no reputation (i.e., a new entrant), the mean of the bidders' signal is \$4.59. As seen in Table 4, the posterior mean of α_1 is 0.46, which is the coefficient parameter of the covariate $\log$(Positive reputation+1) corresponding to the mean parameter μ_1. Therefore, if a seller earns a more positive reputation, the mean of the bidders' signals will increase. This result seems intuitively plausible. The posterior mean of α_2 is -0.03, and α_2 takes a positive value with probability 0.46. Since α_2 is the coefficient parameter of the covariate $\log$(Negative reputation+1) corresponding to the mean parameter μ_1, the number of negative ratings does not have much effect on the mean of the bidder's signal. This result is not intuitively plausible. One possible reason for the tiny effect of negative reputations on the mean of bidders' signals is the positive correlation between positive reputations and negative reputations. The correlation coefficient between positive reputations and negative reputations is 0.86, which represents a high positive correlation. From Table 1, the number of negative ratings is small relative to the number of positive ratings. There are very few auctions in which sellers receive negative ratings. In most cases, sellers receive positive ratings. Many sellers with $\log$(Positive reputation+1)<7 (i.e., sellers with positive reputations, less than 1100 total) had no negative ratings. All sellers with $\log$(Positive reputation+1)≥ 7 had some negative ratings.

Therefore, sellers with more trades receive more (both positive and negative) ratings. From these facts, we conclude that the number of negative ratings does not represent the insincerity of seller but, rather, the abundance of the seller's experience, in our empirical example.

4.2.2　The 22-coin set

Similar to the case of 11-coin set, we assume that the signal S_i follows the normal distribution. That is,

$$S_{it} \sim i.i.d.\ \mathrm{N}(\mu_t, \sigma_t^2),$$

where $\mu_t = \alpha_0 + \alpha_1 X_{t1} + \alpha_2 X_{t2}$, and $\sigma_t^2 = \exp(\beta_0) + \exp(\beta_1) X_{t1} + \exp(\beta_2) X_{t2}$. The parameters $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2)$ are unknown to econometricians. In this empirical illustration, the auction-specific covariates, $X_t = (X_{t1}, X_{t2})$ are the logarithm of "Positive reputation+1" and "Negative reputation+1".

The prior distribution of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $\boldsymbol{\alpha} \sim \mathrm{N}(0, 100\boldsymbol{I})$ and $\boldsymbol{\beta} \sim \mathrm{N}(0, 100\boldsymbol{I})$, where $\boldsymbol{I}$ is the identity matrix of order 3.

Similar to the case of 11-coin set, we use the random walk-based MH algorithm to generate random draws from the posterior distributions. We draw 30000 random samples from the posterior distribution via MH algorithm for each parameter. The burn-in period is 3000.

Table 5 provides the summary of statistics of posterior distributions and the *p*-values of convergence diagnostics for the MCMC (CD) and Inefficiency Factors (IF). All *p*-values of the convergence diagnostics are more than 0.06. Furthermore, the inefficiency factors are less than 188. Therefore, we would obtain the same variance of the posterior sample means from 159 uncorrelated draws, even in the worst case. Figure 3 shows the sample paths of estimated parameters. We conclude that the sample paths of estimated parameters converge to posterior distributions.

Figure 4 shows the posterior densities of parameters for the 22-coin set. Table 6 and Figure 4 provide some posterior inferences. As seen in Table 6, the posterior mean of α_0 is 5.15. Since α_0 is the constant term corresponding to the mean parameter μ. Therefore, when a seller has no reputation (i.e., a new entrant), the mean of the bidders' signal will be \$5.15. The posterior mean of α_1 is 0.70. Since α_1 is the coefficient parameter of the covariate $\log(\text{Positive reputation}+1)$ corresponding to the mean parameter μ, we find that positive reputation has positive effect on the mean of bidders' signals. The posterior mean of α_2 is -0.28 and α_2 takes a positive value with probability 0.30. Recall that

Table 5: The convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF) for the 22-coin set

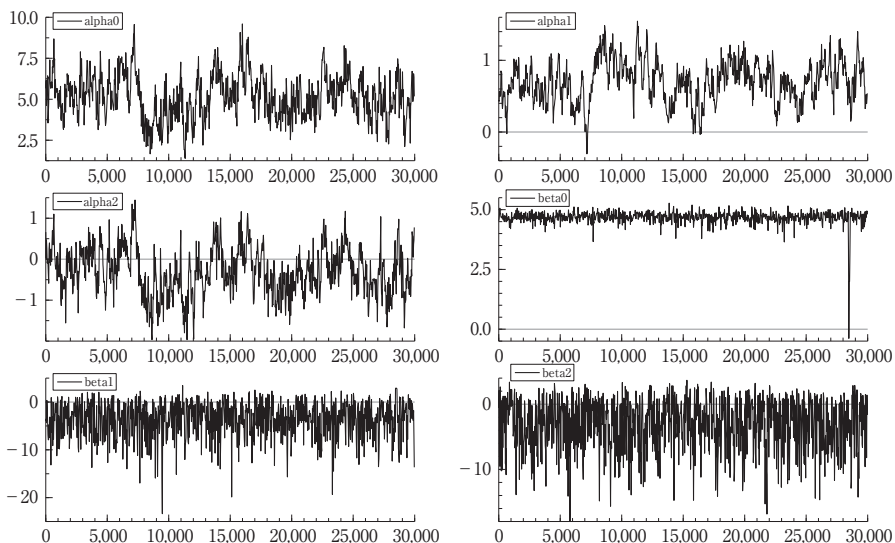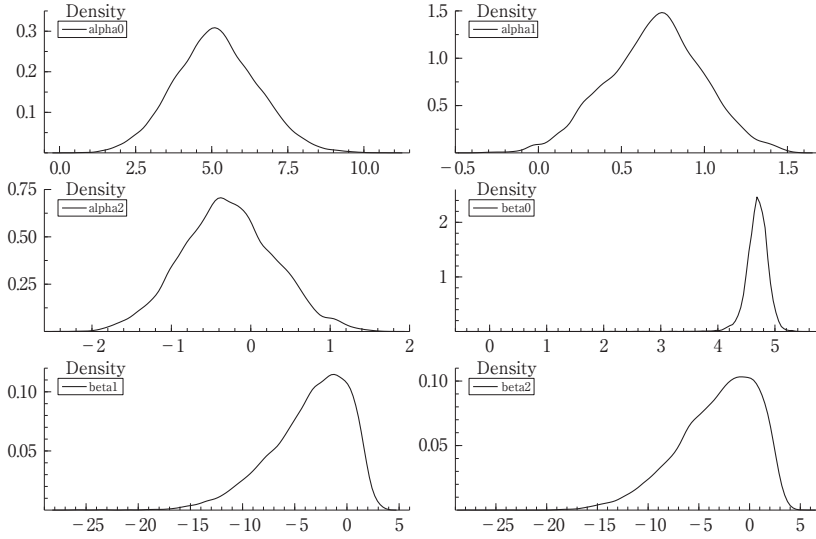| Parameter	Covariate (Coefficient Parameter)	PP	CD	IF
μ_1	Const. (α_0)	1.00	0.19	166.34
	$\log(\text{Pos.Rep.} +1)$ (α_1)	0.99	0.10	187.68
	$\log(\text{Neg.Rep.} +1)$ (α_2)	0.30	0.06	160.56
σ_1^2	Const. (β_0)	1.00	0.85	53.65
	$\log(\text{Pos.Rep.} +1)$ (β_1)	0.18	0.81	21.81
	$\log(\text{Neg.Rep.} +1)$ (β_2)	0.24	0.41	16.99

Figure 3: Sample paths of parameters (22-coin set)

Table 6: Posterior inferences for the 22-coin set

Parameter	Covariate (Coefficient Parameter)	Mean	Stdev.	95% credible interval
μ	Const. (α_0)	5.15	1.36	(2.53, 7.90)
	$\log(\text{Pos.Rep.}+1)$ (α_1)	0.70	0.29	(0.12, 1.25)
	$\log(\text{Neg.Rep.}+1)$ (α_2)	-0.28	0.58	$(-1.42, 0.89)$
σ^2	Const. (β_0)	4.68	0.24	(4.27, 5.02)
	$\log(\text{Pos.Rep.}+1)$ (β_1)	-3.56	3.79	$(-12.37, 1.71)$
	$\log(\text{Neg.Rep.}+1)$ (β_2)	-3.18	3.94	$(-12.21, 2.48)$

Figure 4: Posterior densities (22-coin set)

α_2 is the coefficient parameter of the covariate $\log(\text{Negative reputation}+1)$ corresponding to the mean parameter μ. According to our results, the number of negative ratings does not have much effect on the mean of bidders' signals. This result is not plausible to our intuition. A possible reason is the same as in the case of 11-coin set. That is, a high positive correlation between positive reputations and negative reputations. The correlation coefficient between positive reputations and negative reputations is 0.92, which represents a high positive correlation between positive reputations and negative reputations. Table 2 shows the number of negative ratings is small relative to the number of positive ratings. There are very few auctions in which sellers receive negative ratings. In most cases, sellers receive positive ratings. Analogous to the case of 11-coin set, Many sellers with $\log(\text{Positive reputation}+1) < 7.2$ (i.e., sellers with positive reputations, less than 1330 total) had no negative ratings. All sellers with $\log(\text{Positive reputation}+1) \geq 7$ had some negative ratings. We conclude that the number of negative ratings does not represent the insincerity of seller but, rather, the abundance of the seller's experience. As a result, negative ratings do not have much impact on the mean

of bidders' signals.

5 Counterfactual simulations

In this section, we compute the winner's curse reduction effect in the sense of Chakraborty (2002) and compare the revenue of separate auctions and bundling auctions using the estimated parameters from Section 4.[3]

In our empirical model, the distribution of bidders' signals depends on auction-specific covariates. We compute the winner's curse reduction effect and the expected revenue for a "representative" auction using the sample means of covariates, $\log(\text{Positive reputation}+1)$ and $\log(\text{Negative reputation}+1)$, in Tables 1 and 2 and the posterior mean of the estimated parameters in Tables 4 and 6. The sample means of $\log(\text{Positive reputation}+1)$ and $\log(\text{Negative reputation}+1)$ are

$$\overline{\log(\text{Positive reputation}+1)} = 6.77 \text{ and } \overline{\log(\text{Negative reputation}+1)} = 1.34,$$

respectively. The number of participants for a representative auction is $N=7$. Subsequently, the bidding functions can be computed using equations (1) and (2).

In our empirical example, since the separate items, $k=1$ and $k=2$, are the same item, we cannot estimate the parameters for item $k=2$ directly. In other words, we cannot obtain the estimates for coefficient parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ for item 2 from observed bids. However, for an arbitrary fixed covariates (and hence for the representative auction), the distribution of bidders' signals for item 2 can be identified. Since bidder i's private signal for item $k=1$, S_{1i}, and bidder $i's$ private signal for item $k=2$, S_{2i}, are independent, the distribution of bidders' signals for item 2 can be recovered from the identified distributions of bidders' signals for item 1, S_{1i}, and bidders' signals for bundled item, S_i. Note that while we assume the independence, we do not assume that S_{1i} and S_{2i} have identical distributions.

Since our parametric specification imposes that S_{1i}, S_{2i}, and S_i are normal random variables, from the reproductive property of normal distributions, we have $S_{2i} \sim \mathrm{N}(\mu-\mu_1, \sigma^2-\sigma_1^2)$, where (μ, σ^2) and (μ_1, σ_1^2) are the parameters for distributions of S_i and S_{1i}, respectively. Let (μ_2, σ_2^2) be the parameter vector for distributions of S_{2i}. By the estimated parameters and the sample mean of covariates, we gain $\mu_2=1.85$ and $\sigma_2^2=40.53$. Note that, since the mean of the signals for item 1 is $\mu_1=7.66$, $\mathrm{E}(S_{1i}) > \mathrm{E}(S_{2i})$ holds. This inequality seems intuitively plausible because the willingness to pay for the second item is usually less than that for the first item.

The statement of Theorem 1 is the winner's curse reduction effect, as proposed by Chakraborty (2002). Namely, $b_i(s) \geq b_{1i}(s_1) + b_{2i}(s_2)$. For each fixed signal $s=5, 10, 15$, varying the value of the signal for item 1, s_1, from 3.0 to s, we compute $b_i(s) - (b_{1i}(s_1) + b_{2i}(s_2))$.

Figures 5, 6, and 7 report the difference of the bidding function $b_i(s) - (b_{1i}(s_1) + b_{2i}(s_2))$ for

fixed signals $s = 5, 10, 15$, respectively. The shape of the graph with $s = 5$ is not similar to that of the graphs with $s = 10, 15$. When $s = 5$, the difference decreases as the signal for item 1, s_1, increases. On the other hand, when $s = 10$ and 15, the difference decreases for $s_1 \in (3.0, 8.0)$ and $s_1 \in (3.0, 11.0)$ and it increases for $s_1 > 8.0$ and $s_1 > 11.0$, respectively. The values of $b_i(s) - (b_{1i}(s_1) + b_{2i}(s_2))$ for $s = 5, 10, 15$ are similar, around \$2.50.

One may mistakenly conclude that Theorem 1 implies the revenue of bundling auctions is higher than that of separate auctions. Actually, Theorem 1 does not imply revenue ranking. In Theo-

Figure 5: Difference of the bidding function with *s*=5
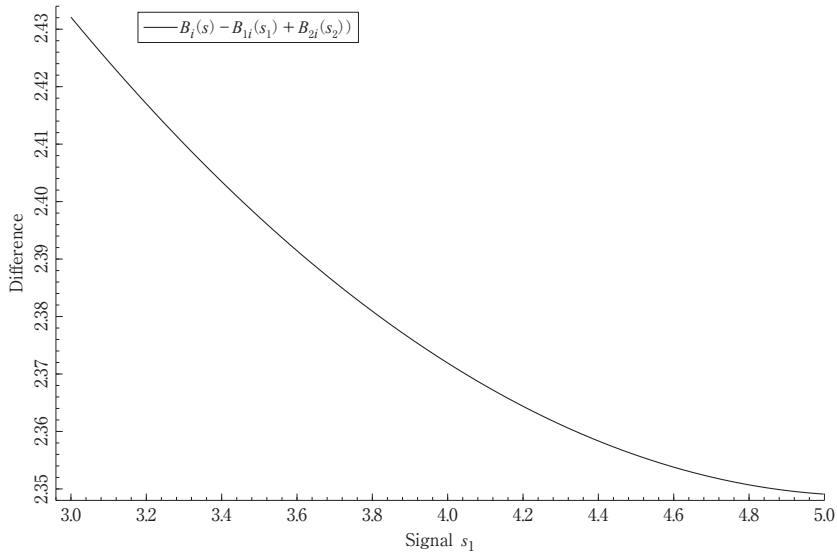
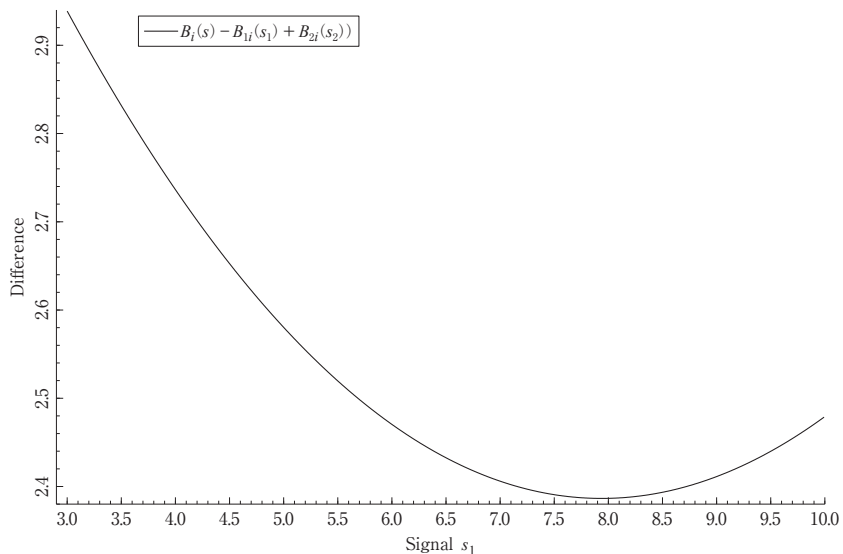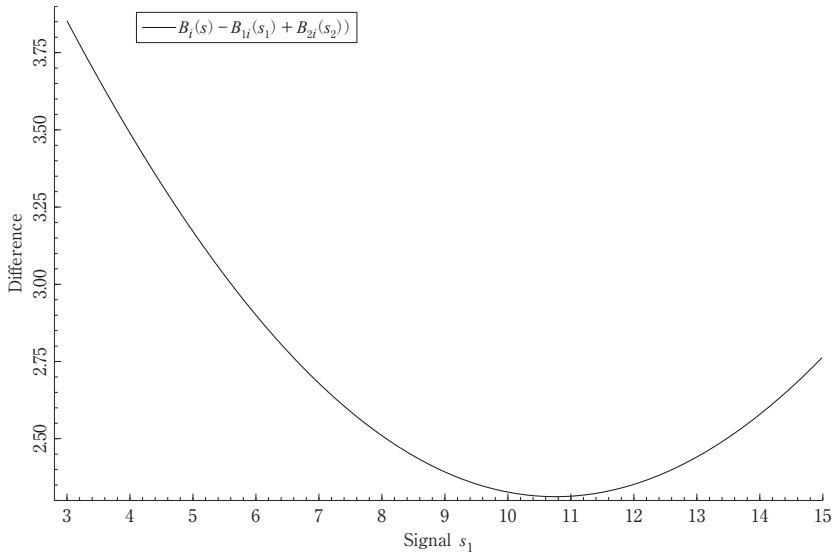**Figure 6: Difference of the bidding function with *s*=10**

Figure 7: Difference of the bidding function with s=15

rem 1, for any signal of bundling auctions, $S_i=s$, the equation $s=s_{1i}+s_{2i}$ must hold. When we compare the revenues, the equation $s=s_{1i}+s_{2i}$ need not hold. The realizations of S_{1i} and S_{2i} are determined independently.

The expected revenues are computed by the Monte Carlo simulation method. Using the estimated parameters and the sample means of covariates, we draw the signals of bundling and separate auctions from the estimated distributions. We assume that the number of potential bidders is N=7. Then, the equilibrium bids for signals are computed via equation (1) and (2). The winning bids are the second-highest bids for both bundling and separate auctions. The revenue difference is computed by the difference between the bundling auction's winning bid and that of the separate auction. We iterate this procedure 5000 times.

The density of revenue differences between bundling and separate auctions is shown in Figure 8. The shape of the density is symmetrical at point 0. Table 7 reports the summary statistics of revenues and revenue differences. In Table 7, "Mean" and "Stdev." are the mean and the standard deviation of revenue differences, respectively. Similarly, ".25 quantile," "Median," and ".75 quantile" represent the first quartile, the second quartile, and the third quartile. The probability that the revenue of bundling auctions is higher than that of separate auctions is denoted by "PP."

According to Figure 8 and Table 7, the revenue of bundling auctions is higher than that of separate auctions with probability 0.53. The expected revenue difference is $0.37. Therefore, sellers can gain an additional profit of $0.37 by using a bundle auction rather than two separate auctions. Since the average transaction price of bundled items (22-coin sets) is $8.98, we find that the value of additional gains are not negligible. In the theoretical literature, Chakraborty (2002) discussed the

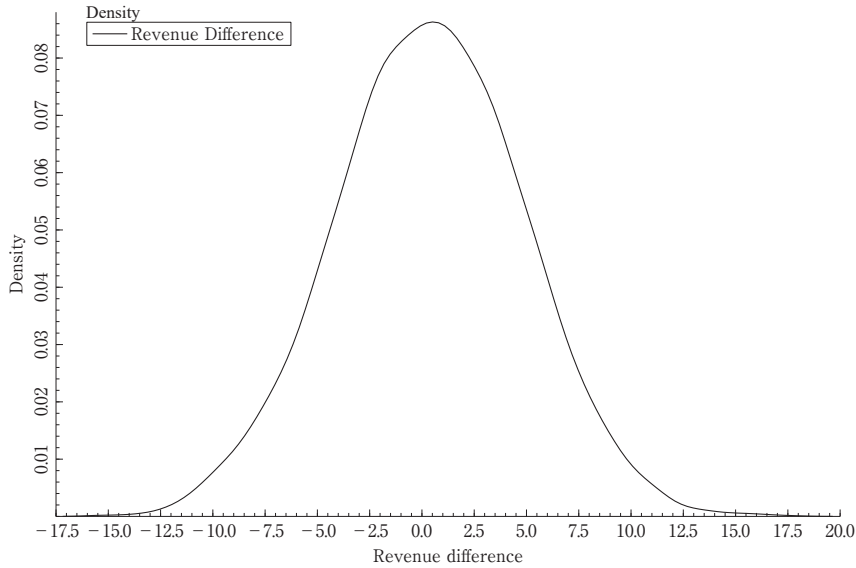**Figure 8: Density of revenue difference between bundling auctions and separate auctions**

Table 7: Summary statistics of revenue and revenue differences

	Mean	Stdev.	.25 quantile	Median	.75 quantile	PP
Revenue (bundle)	8.67	3.12	6.61	8.85	10.88	—
Revenue (item 1)	6.99	2.48	5.39	7.10	8.73	—
Revenue (item 2)	1.32	1.90	0.09	1.41	2.67	—
Revenue difference	0.37	4.47	−2.60	0.38	3.40	0.53

revenue ranking between the revenue of bundling auctions and separate auctions. He found that bundling auctions generate more expected revenue than do separate auctions when the number of bidders is sufficiently small. According to Tables 1 and 2, the number of participants at most 7. Therefore, our empirical example does not contradicts the result of Chakraborty (2002).

6 Conclusions

In this paper, we focused on bundling auctions in online auction markets. In online auction markets (e.g., eBay and Yahoo!), sellers often sell two or more items in bundling auctions. Conversely, other sellers sell the same items separately. We propose an estimation procedure for bundling auction models within the pure common value paradigm.

Our empirical example is eBay mint coin set auctions in 2014. In our data set, there are two kinds of coin sets: 11-coin sets and 22-coin sets. We regard the 11-coin sets as the separate item and the 22-coin set as the bundled item. We also conducted some counterfactual simulations using the estimated parameters. We computed the winner's curse reduction effect following Chakraborty (2002)

precedent and compared the revenue of bundling auctions and separate auctions. We found that the value of the winner's curse reduction effect is about $2.5. For revenue comparison, we found that the expected revenue in the bundling auctions is higher than that in the separate auctions by $0.37. Since the average transaction price of bundled items（22-coin sets）is $8.98, the value of additional gains are not negligible.

There are some avenues for future research in this paper. For one, we ignored the endogenous entry of bidders. In general, bidders will decide endogenously to participate, whether in bundling auctions or separate auctions. Analogously, we also ignored the seller's incentive to decide which item（the bundled item or separate items）to sell. The seller's decision as to which item to sell will depend on the revenue ranking between bundling auctions and separate auctions.

Acknowledgment

Notes

1 This specification is the special case of Chakraborty（2002）and has been used in several papers. Example are Goeree and Offerman（2002）for auctions within common value and private values paradigm and Shahriar（2008）and Shahriar and Wooders（2011）for auctions with buy prices model.

2 We omit subscript $k=1$ for the coefficient parameters α and β.

3 Chakraborty（2002）also discussed the expected revenues of both bundling auctions and separate auctions. Under the regularity conditions that are satisfied in our parametric specifications（i.e., normally distributed signals）, He found that revenue ranking between the revenue of bundling auctions and separate auctions depends on the number of potential bidders, N. He found that bundling auctions generate more expected revenue than do separate auctions for all $N<N^*$, where N^* is a sufficiently small number.

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