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Realized jump beta: Evidence from high-frequency data on Tokyo Stock Exchange

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Abstract

This study measures the realized jump beta of sector portfolios constructed from Japanese high-frequency intraday data to assess the dynamics in jump beta aggregated over fixed intervals of time. When we test the null hypothesis that jump beta remains constant over time, the result strongly rejects the constancy of annually aggregated jump beta, but does not frequently reject that of monthly jump beta. Given the worldwide evidence of fractional integration in realized variance and covariance, the estimation result under the assumption of a pure fractional noise process of the monthly jump beta indicates a smaller average degree of integration, namely *ARFIMA*(0, 0.2, 0), than that of a total realized beta largely including diffusive risk component of asset returns. Moreover, the jump beta might be naturally modeled as a stationary $I(0)$ process.

JEL Classification Numbers: G12, G15.
Keywords: Jump beta, Market jump variation, Jump covariation, High-frequency data.

1. Introduction

In the simple one-factor capital asset pricing model (CAPM), the systematic risk factor loading is market beta determined by the covariance of asset and market portfolio returns. A key question in this framework is whether or not market beta is constant over fixed intervals of time. From theoretical and empirical perspectives, studies such as Hansen and Richard (1987), Ferson et al. (1987) and Jagannathan and Wang (1996) suggest that market beta is likely to vary with condition-

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ing variables. Within the framework of realized variances and covariances based on the high-frequency intraday data first popularized by Andersen and Bollerslev (1998), Andersen et al. (2006) analyze the dynamics of realized market beta, or equivalently nonparametric period-by-period beta, defined as the realization of the ratio of integrated asset and market covariation to integrated market variation of the underlying price processes.

The development in financial econometrics literature based on the high-frequency data sheds light on large jumps of asset and market return processes (e.g., Andersen et al. 2007; Barndorff-Nielsen and Shephard 2004; Huang and Tauchen 2005; Lee and Mykland 2008). A classical asset pricing model assumes that jump risk for individual stock is non-systematic, so that the corresponding jump beta is zero. However, the observed cross-correlations for extreme asset returns in the financial market imply that the jump beta is not negligible and the market beta could be decomposed into systematic diffusive and jump risk components. The presence of substantial jump risks is also supported by the existing positive price against the expected variation of the underlying asset jump return in derivatives markets; in fact, the corresponding option-implied downside jump variation could be a significant predictor for future equity risk premium (e.g., Pan 2002; Eraker 2004; Bollerslev and Todorov 2011; Bollerslev et al. 2015; Andersen et al. 2020).

To my knowledge, there is limited research on jump beta based on high-frequency data. Todorov and Bollerslev (2010) propose an estimator of jump CAPM beta based on higher-order power variations. They find that the estimates differ statistically from diffusive CAPM beta for some of the U.S. stocks. Mancini and Gobbi (2012) propose an estimator of jump beta based on the ratio of truncated jump covariation to jump variation. Li et al. (2017) propose a test statistic for time-varying jump beta. The empirical evidence has been provided only in the U.S. financial market such as by Bollerslev et al. (2016) and Alexeev et al. (2016, 2017). For instance, Bollerslev et al. (2016) show that the jump betas of 1,000 U.S. stocks entail significant risk premiums and remain significant even after controlling for firm characteristics and other explanatory variables. Overall, they suggest the usefulness of separately considering the two different types of components in market beta.

The main purpose of our study is to understand the dynamics of the realized jump beta constructed based on the high-frequency intraday data and to provide new evidence from the Japanese financial market. First, we investigate how much sensitivity of major sector portfolios is captured by the jump beta at the moment when large jumps happen to the market portfolio. To measure the jump beta, we detect the 10-minute intraday market jump returns based on the truncation method first popularized by Mancini (2001, 2009), and construct the ratio of jump covariation between market jump and corresponding sector portfolio returns to market jump variation aggregated over fixed intervals of time. We find that annually aggregated jump beta obtained by pooling the 10-minute jump returns every year is nonzero and that the dynamics is very smooth, but allowing for the jump beta to change on monthly frequency captures more clearly the large changes in the sensitivity of sector portfolios to the market jumps.

We then examine the time-varying jump beta over years and months using the test statistics of Li et al. (2017). If the test statistic rejects the null hypothesis that jump beta remains constant over fixed intervals of time, we investigate the time-series property of jump beta, such as the degree of persistence. The test shows that the constancy of annually aggregated jump beta is always rejected at the 1% significance level for all sector portfolios used in our study, while that of monthly aggregated jump beta is not rejected for about half of the monthly samples. This result suggests that it would be more reasonable to investigate the time-series property of realized jump betas at a monthly frequency. Given the worldwide evidence of fractional integration in realized variance and covariances, we apply a pure fractional noise process to monthly jump betas. The estimation result shows a small average degree of fractional integration such that the estimates of the long-memory parameter is around 0.2. It is less persistent than the total realized betas and covariances, typically with the degree of integration around 0.4 in our samples. Furthermore, we fit the autoregressive integrated moving average (ARIMA) process to the realized jump beta, and the estimation result suggests that the realized jump beta might be naturally modeled as a stationary $I(0)$ process.

The remainder of this article is organized as follows. Section 2 provides a brief explanation of the procedure for calculating realized jump beta used throughout the study. Section 3 describes the data that we use in the empirical analysis and presents the empirical results for time-varying realized jump beta. Section 4 concludes.

2. Measuring Realized Jump Beta

Realized jump beta is defined as the realization of covariation between market jump return and the contemporaneous asset return divided by market jump return variation. To calculate the quantity, we will first detect the high-frequency market jump returns based on the truncation method proposed in Mancini (2001, 2009). For the simplicity of notation, the daily interval $[t-1, t]$ is divided into non-trading $[t-1, t-1+\pi]$ and trading $[t-1+\pi, t]$ intervals. We then define equally-spaced high-frequency market returns as $r_{i,t} = f_{t-1+\pi+i\Delta m} - f_{t-1+\pi+(i-1)\Delta m}$ for $i = 1, \ldots, m$, $t = 1, \ldots, T$, where $f_.$, m, and T are the log market price, the number of intraday market returns in each day, and the number of days, respectively. The interval length of market returns is denoted as $\Delta_m = \frac{(1-\pi)}{m} = \frac{(1-\pi)T}{mT} = \frac{(1-\pi)T}{n} = \Delta_n$, where n is the number of market returns within the fixed time interval $[0, T]$. We examine high-frequency data on equally spaced series at 10-minute intervals of returns in our calculation of the realized jump beta.

We consider the time-varying threshold value for the identification of market jump returns as introduced in Bollerslev and Todorov (2011), Todorov and Tauchen (2012), Andersen et al. (2015), and Li et al. (2017). The idea is that if an absolute value of intraday market return exceeds the threshold value, the corresponding market jump return is identified. The computation of the time-varying threshold level considers the intraday periodicity in the return volatility because the deterministic diurnal pattern in volatility is well observed in financial markets. One of the statistics measuring the periodicity is the time-of-day (TOD) factor,

$$TOD_i = NOI_i \times \frac{\sum_{t=1}^{T} r_{i,t}^2 1_{\{|r_{i,t}| \leq \bar{v}\Delta_m^{\omega}\}}}{\sum_{t=1}^{T} \sum_{j=1}^{m} r_{j,t}^2 1_{\{|r_{j,t}| \leq \bar{v}\Delta_m^{\omega}\}}}, \text{ for } i = 1, \cdots, m. \tag{1}$$

Theoretically, $\omega \in (0, 0.5)$ will work, and we fix $\omega = 0.49$ as with the previous studies. The constant truncation level $\bar{v}$ is calculated as $3 \times \overline{BV}^{1/2}$, where $\overline{BV}$ is the sample average of open-to-close bipower variation measure BV_t^{open} over $[0, T]$ proposed in Barndorff-Nielsen and Shephard (2004), that is, $\overline{BV} = 1/T \sum_{t=1}^{T} BV_t^{open}$. The numerator and denominator in (1) estimate the continuous or diffusive return variation for each i-th intraday interval over $[0, T]$ and the total continuous return variation, respectively. NOI_i is expressed as $\frac{\sum_{t=1}^{T} \sum_{j=1}^{m} 1_{\{|r_{j,t}| \leq \bar{v}\Delta_m^{\omega}\}}}{\sum_{t=1}^{T} 1_{\{|r_{i,t}| \leq \bar{v}\Delta_m^{\omega}\}}}$ and adjusts the different number of continuous returns for each interval i.

We use the time-varying threshold value, denoted as $v_{i,t}$, which depends on the intraday periodicity and time-varying volatility across days,

$$v_{i,t} = c \times \sqrt{\frac{\min(RV_t^{open}, BV_t^{open})}{1 - \hat{\pi}}} \times TOD_i \times \Delta_m^{\omega}, \tag{2}$$

where $\min(RV_t^{open}, BV_t^{open})$ is a function that selects the smaller value from open-to-close realized variance and bipower variation measure. $\hat{\pi}$ is the ratio of close-to-open return variation on close-to-close return variation over $[0, T]$ expressed as $\frac{\sum_{t=1}^{T} (f_{t-1+\pi} - f_{t-1})^2}{\sum_{t=1}^{T} (f_t - f_{t-1})^2}$. In the Japanese stock market, the numerator of $\hat{\pi}$ is calculated as the summation of squared overnight and lunchtime returns over $[0, T]$. The constant scaling factor c is set as 4 in our calculation. The time-varying threshold value $v_{i,t}$ increases as daily market return volatility increases. Therefore, an absolute value of intraday market returns $|r_{i,t}|$ needs to exceed the large threshold value to be judged as jump. The collection of high-frequency market jump returns is denoted as $(\Delta_j^n Z)_{j \in \mathcal{L}_n(\mathcal{D})}$, where $\mathcal{L}_n(\mathcal{D})$ denotes indices of intervals that contain the market jumps in the region $\mathcal{D}$. Let $(\Delta_j^n Y)_{j \in \mathcal{L}_n(\mathcal{D})}$ be a collection of contemporaneous returns of an asset over the times of market jumps and $\Delta_j^n X$ be the 2 by 1 vector of $\Delta_j^n Y$ and $\Delta_j^n Z$, Li et al. (2017) introduce a simple sample analogue estimator of jump covariation matrix $Q_n(\mathcal{D})$,

$$Q_n(\mathcal{D}) = \begin{pmatrix} Q_{YY,n}(\mathcal{D}) & Q_{YZ,n}(\mathcal{D}) \\ Q_{ZY,n}(\mathcal{D}) & Q_{ZZ,n}(\mathcal{D}) \end{pmatrix} = \sum_{j \in \mathcal{L}_n(\mathcal{D})} \Delta_j^n X \Delta_j^n X^\top. \tag{3}$$

Finally, realized jump beta can be calculated as the ratio of realized jump covariation estimator between $\Delta_i^n Y$ and $\Delta_i^n Z$ to realized market jump variation estimator,

$$\beta_n^J = \frac{Q_{YZ,n}(\mathcal{D})}{Q_{ZZ,n}(\mathcal{D})}. \tag{4}$$

We should note that the estimator in (4) corresponds to the no weighting version of the optimally weighted estimator of the jump beta proposed by Li et al. (2017). The calculation of weights requires the pre-jump and the post-jump spot covariance matrices. We do not use the weighted estimator in our calculation of jump beta because of the shortage of the corresponding pre- and post-jump continuous returns while calculating the weight for the jump return identified at the start- and end-time intervals in open-to-close markets.

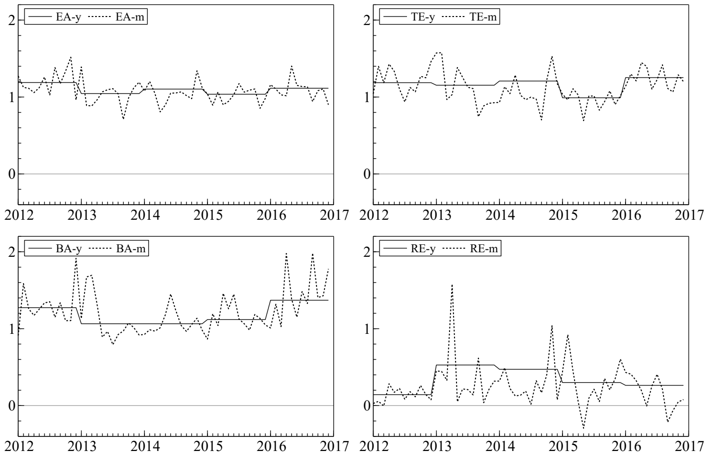
3. Empirical Results

3-1. Data

In our empirical analysis, we construct jump betas of four sector portfolios for electrical appliances (EA), transportation equipment (TE), banks (BA) and real estate investment trust (RE). We proxy market portfolio in Japan with Tokyo stock price index (TOPIX) which is a value-weighted index based on all domestic common stocks in the first section of the Tokyo Stock Exchange. To avoid a market microstructure effect due to ultra high-frequency return series, which is induced by various market frictions such as the discreteness of price changes and bid-ask bounces, inter alia, we rely on the 10-minute high-frequency returns of the sector and market portfolios obtained by Nikkei NEEDS-TICK data from January 2012 to December 2016. The equity market timings are 9:00–11:30 and 12:30–15:00 in our sample period, and thus the number of intraday intervals in a day is $m = 30$.

Figure 1 plots annually and monthly aggregated realized jump betas of the four sector portfolios. The annually estimates in solid lines, which are obtained by pooling the 10-minute market jump and corresponding sector portfolio returns every year, are very smooth. The jump betas over annual frequencies tend to be a little higher than 1 for EA, TE, and BA, but much lower than 1 for RE. The monthly estimates obtained by pooling the 10-minute jump returns every month are reported in dot-

Figure 1: Time-series plot of annually and monthly aggregated realized jump betas



Note: The solid and dotted lines represent the estimates of annually and monthly aggregated realized jump betas of four sector portfolios constructed from the 10-minute high-frequency returns of market jump and the contemporaneous sector portfolios from 2012 to 2016.

ted lines. Allowing for jump beta to change monthly, we can capture noisier and higher fluctuations of jump betas relative to annual jump betas. The estimates for EA, TE, and BA occasionally take below one, and negative estimates for RE occur very rarely.

As noted in Li et al. (2017), the realized jump beta is consistent with a slope coefficient when we conduct linear regression without a constant term of sector portfolio returns on market jump returns aggregated over fixed intervals of time. To illustrate the jump regressions, Figure 2 displays the scatter plots of 10-minute market jumps and the respective sector portfolio returns, together with their linear fits. We find that the monthly aggregated jump regression provides a reasonably better fit relative to the annually aggregated jump regression. The results motivate us to explore the test of constant jump beta over different frequencies.

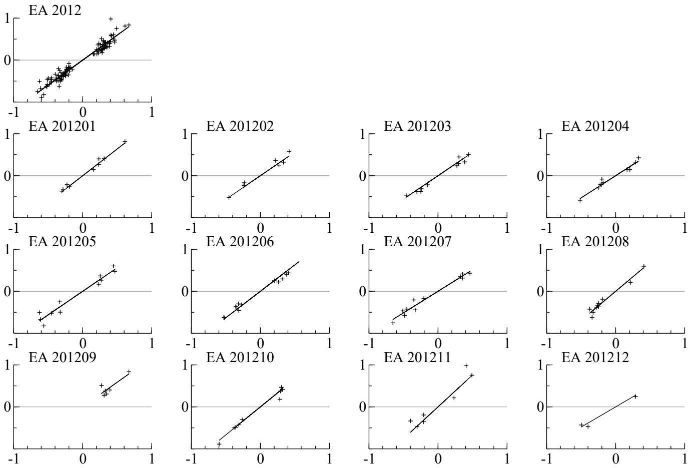
3-2. Test for Constant Jump Betas

We apply a test statistic for constant jump betas proposed by Li et al. (2017). The test is based on the fact that the constant linear jump regression model over fixed intervals of time is equivalent to the singularity of the realized jump covariation matrix. The null hypothesis that the jump beta remains constant over time is rejected when the determinant of a sample analogue estimator of the jump covariation matrix is larger than a critical value,

$$\Delta_n^{-1}\det[Q_n(\mathcal{D})] > \mathcal{CV}_n^{\alpha}. \tag{5}$$

$\mathcal{CV}_n^{\alpha}$ is the critical value at the significant level $\alpha \in (0, 1)$. The test statistics can be also written in terms of realized jump correlation,

Figure 2: An illustration of linear jump regressions with scatter plots and linear fits



Note: The x-and y-axes represent the 10-minute returns of market jump and the contemporaneous electric appliance sector portfolio in 2012. The first and the other panels show linear jump regressions over a year and every month, respectively.

$$1-\rho_n^2(\mathcal{D}) > \frac{\Delta_n \mathcal{CV}_n^{\alpha}}{Q_{ZZ,n}(\mathcal{D})\, Q_{YY,n}(\mathcal{D})}, \tag{6}$$

where $\rho_n(\mathcal{D})$ is the realized jump correlation expressed as $\frac{Q_{YZ,n}(\mathcal{D})}{\sqrt{Q_{YY,n}(\mathcal{D})Q_{ZZ,n}(\mathcal{D})}}$. The computation of the critical value $\mathcal{CV}_n^{\alpha}$ is based on Monte Carlo simulations because the null asymptotic distribution is nonstandard, and its conditional $(1-\alpha)$ quantiles cannot be expressed in closed form. We simulate a large number of Monte Carlo simulations to calculate the $(1-\alpha)$ quantiles of copies of the limiting variable using pre-jump and post-jump spot covariance matrices, realized jump beta, and uniformly distributed on the unit interval and bivariate standard normal variables (See Algorithm 1 and Theorem 1 in Li et al. (2017) for more details). Then, we compute the p-value for inference.

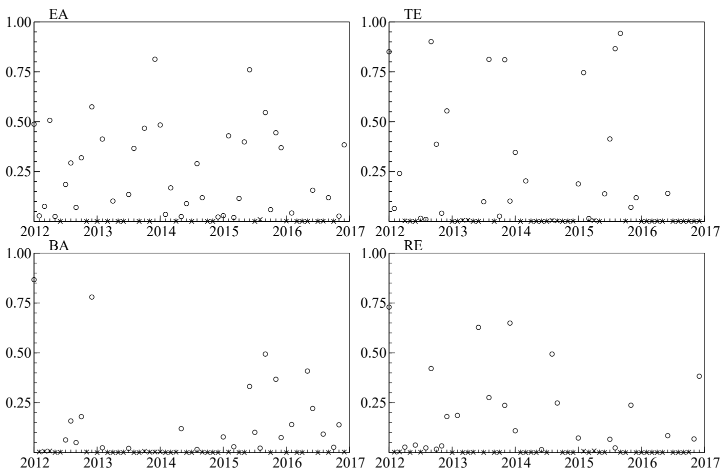
Table 1 reports annually aggregated realized jump betas of four sector portfolios and the p-values of the test statistics (5) or (6) in parentheses. We also report the numbers of 10-minute market (TOPIX) jump returns in Table 1, which range from 107 in 2012 to 248 in 2016. In 2012 and 2016, the jump betas deviate upward from 1 for the EA, TE, and BA sectors and deviate downward from 1 for the RE sector. Despite the apparent variation of year-by-year jump betas, the null hypothesis that annually aggregated jump beta remains constant is strongly rejected in all cases. The result shows the deviations from linearity in the annually aggregated jump regressions and the possibility that the jump beta changes over a shorter period of time. This might be common with the implications of the conditional capital asset pricing models. As opposed to the annual basis, we test the constancy of monthly aggregated jump betas. Figure 3 plots time-series of p-values of the test statistics

Table 1: Annually aggregated jump beta and test for the constancy

	2012	2013	2014	2015	2016
EA-y	1.19	1.05	1.10	1.04	1.11
p-value	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)
TE-y	1.19	1.15	1.21	0.99	1.25
p-value	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)
BA-y	1.27	1.06	1.06	1.12	1.37
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
RE-y	0.14	0.53	0.47	0.30	0.26
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Number of 10-minute jump returns of TOPIX within a year					
	107	207	150	160	248

Note: The table reports the estimates of annual jump betas for four sector portfolios and the *p*-values for the constancy test of the jump beta in parenthesis. The last row reports the number of 10-minute market (TOPIX) jump returns every year.

Figure 3: P-values for the constancy test of monthly aggregated jump beta



Note: The circle and cross symbols represent *p*-values for the null hypothesis of the constant monthly aggregated jump beta, which are more than and less than 0.01 , respectively. The sample period covers the period from January 2012 to December 2016.

(5) or (6) from January 2012 to December 2016 (60 months), where the circle (cross) symbol means that the *p*-values exceed (fall below) the 1% significance level. We find that the null hypothesis that monthly jump beta remains constant is not rejected even at the 1% significance level for about half of the monthly samples. These results suggest the linearity of the jump regression model over periods of months and the stability of monthly realized jump beta.

3-3. Time-Series Property of Monthly Jump Betas

We will investigate the time-series property of monthly aggregated jump beta based on the results in the previous subsection. Rows two to five in Table 2 report the augmented Dickey-Fuller unit root test with intercept and different augmentation lags up to four. The test rejects the null hypothesis of unit root for the jump betas of EA, TE, and RE sectors with all augmentation lags and for BA sector with the first augmentation lag. We also represent the Ljung-Box portmanteau test statistics for up to 12th-order autocorrelation in the last row of Table 2. The null hypothesis of no autocorrelation up to 12 lags is rejected at 5 or 10% significance levels only for jump betas in two out of four sector portfolios. In fact, the values of $LB(12)$ seem to be quite low, compared with the stylized fact of highly significant asset return volatility clustering. This result indicates that the realized jump betas are totally less persistent. This is confirmed by the quite low and moderately declining sample autocorrelation and partial autocorrelations reported in Figure 4.

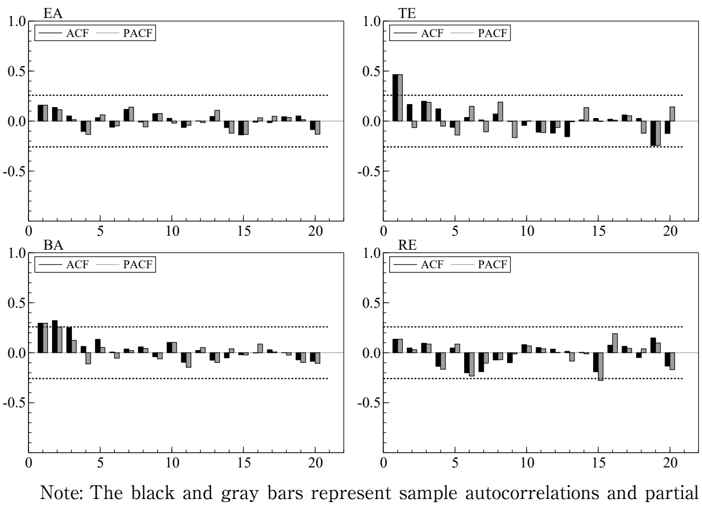
Many authors suggest that asset return volatilities are well-described by a fractional noise process with the degree of fractional integration such that the long-memory parameter d is around

Table 2: Test for the dynamics of monthly aggregated jump beta

	EA	TE	BA	RE
$ADF(1)$	-4.28^{***}	-4.13^{***}	-2.96^{**}	-4.76^{***}
$ADF(2)$	-3.58^{***}	-2.88^{*}	-2.26	-3.63^{***}
$ADF(3)$	-3.58^{***}	-3.00^{**}	-2.28	-3.67^{***}
$ADF(4)$	-2.96^{**}	-3.09^{**}	-1.76	-2.90^{*}
$LB(12)$	5.74	21.91^{**}	19.73^{*}	10.48

Note: $ADF(i)$ denotes the augmented Dickey-Fuller unit root test with intercept and with i augmentation lags. $LB(12)$ denotes the Ljung-Box portmanteau statistic for up to the 12th-order autocorrelation. The superscripts ***, **, and * indicate significance at the 1, 5, and 10% levels, respectively.

Figure 4: Sample autocorrelation and partial autocorrelation of monthly jump beta



Note: The black and gray bars represent sample autocorrelations and partial autocorrelations, respectively, for monthly jump betas of four sector portfolios.

0.4, indicated by Andersen et al. (2001) for spot exchange rate markets and by Andersen et al. (2003) for the U.S. equity market, and around 0.5, indicated by Ubukata and Watanabe (2014) for the Japanese equity market. Andersen et al. (2006) propose the total realized beta including systematic diffusive and jump risk components, which is defined as realized covariance between asset and market returns divided by realized market variance, and find that the total realized betas are less persistent in the U.S. equity market. Despite the empirical evidence of realized covariances and beta, very little research exists on the presence of long-memory in realized jump beta. We apply the simplest autoregressive fractionally integrated moving average model, $ARFIMA(0, d, 0)$, for the time-series of monthly aggregated jump beta to focus on the long-memory parameter d. Table 3 shows the estimation results of the $ARFIMA(0, d, 0)$ model for jump betas of the four sector portfolios. The estimates of the long-memory parameter are lower than 0.4 and the average degree of fractional integration is 0.2. The results indicate very weak long-memory property in the monthly aggregated jump beta series. We will compare the jump beta and covariation with the total beta and covariation in the next subsection.

As motivated by the empirical finding of low-order fractional integration, we conduct a test of short or long-range dependence in Table 4, which reports the classical Hurst-Mandelbrot rescaled range statistics with $q = 0$ and Lo's (1991) modified rescaled range statistics with $q \geq 1$. Judging from

Table 3: Estimation result of $ARFIMA(0, d, 0)$ model for monthly aggregated jump beta

	EA	TE	BA	RE	Mean
d	0.13	0.33	0.27	0.08	0.20
s.e.	0.00	0.00	0.00	0.00	

Note: The parameter d and s.e. represent the degree of fractional integration and standard error of the estimates. The last column reports the average of the parameter estimates for four sector portfolios.

Table 4: Rescaled range test statistics of monthly aggregated jump beta

	EA	TE	BA	RE
$V_{q=0}$	1.659	2.206**	2.278**	1.396
$V_{q=1}$	1.541	1.822**	2.002**	1.310
$V_{q=2}$	1.453	1.677*	1.797**	1.268
$V_{q=3}$	1.402	1.575	1.657*	1.225
$V_{q=4}$	1.394	1.504	1.578	1.227
$V_{q=5}$	1.383	1.469	1.516	1.220
$V_{q=6}$	1.383	1.442	1.475	1.243
$ACF(1)$	0.159	0.465	0.295	0.135
q^*	2	5	3	1

Note: $V_{q=0}$ and $V_{q\geq1}$ are the classical and modified rescaled range statistics. The superscripts ** and * indicate significance at the 5 and 10% levels. The 5 and 10% critical values are 1.747 and 1.620, respectively. q^* is the lag chosen by Andrews' (1991) data-dependent formula.

the statistics with the optimal number of lags q^* chosen by Andrews' (1991) data-dependent formula, the null hypothesis of no long-range dependence in monthly realized jump betas of EA, TE, and RE sector portfolios is not rejected even at the 10% significance level. In light of the result that the jump beta does not exhibit long-range dependence, we are interested in the alternative where the realized jump betas are $I(0)$ processes. To explore this possibility, we fit the ARIMA model to the realized jump betas, with lag selected by the Akaike information criterion (AIC). Table 5 reports the ARIMA model selection and estimation results for the monthly jump beta in each sector portfolio. The selected models are $ARIMA(0, 0, 0)$ for EA and RE, $ARIMA(1, 0, 0)$ for TE, and $ARIMA(2, 0, 0)$ for BA. The estimation results suggest that the time series of jump beta might be within the context of stationary $I(0)$ processes.

3-4. Comparison between Monthly Jump Betas and Total Betas

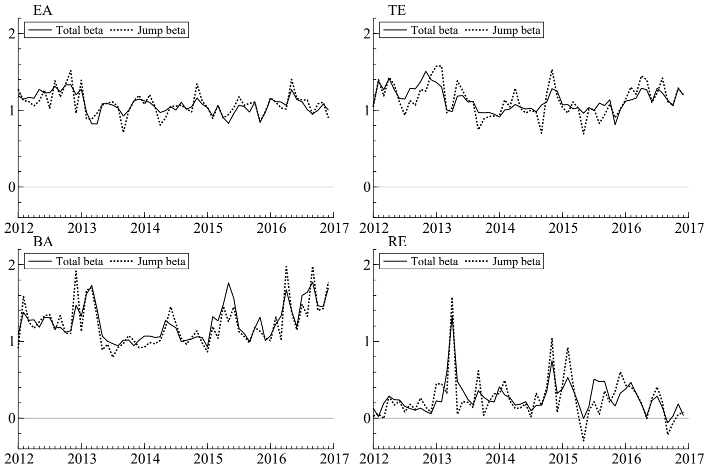
In this subsection, we compare the time-series properties of the realized jump betas and the total realized beta before being decomposed into systematic diffusive and jump risk components. The sample correlation coefficients between the two betas of four sector portfolios are around 0.8. Despite the relatively high correlation, the time-series plot of monthly aggregated jump and total betas reported in Figure 5 reveals the difference between the two more clearly. In particular, the jump risk component of the market beta (dotted line) looks more volatile than the total beta. Furthermore, the more prominent difference is the degrees of fractional integration. Table 6 reports the estimation results of the long-memory parameter d in the $ARFIMA(0, d, 0)$ model for monthly jump beta (second row), market jump variation and covariation for sector portfolios (third row), and the total ones (fourth and fifth rows). We find that the estimates of the long-memory parameter d for total beta and (co)variation are on average 0.43 and 0.36 larger than those of jump beta and (co)variation, which are 0.20 and 0.24, respectively. The results suggest that the total betas and (co)variation are highly persistent relative to the jump betas and (co)variation. Thus, we conclude that separately

Table 5: ARIMA model selection and estimation results

	Constant	AR(1)	AR(2)	AIC	Inverted Roots	
EA	1.08 (0.02)			−53.45		
TE	1.13 (0.04)	0.46 (0.11)		−28.64		
BA	1.22 (0.06)	0.23 (0.13)	0.28 (0.13)	12.98	0.66	−0.43
RE	0.25 (0.04)			22.76		

Note: The table reports estimation results of the fitted ARIMA model selected by AIC for monthly jump beta in each sector portfolio. The sample covers the period from January 2012 to December 2016 for a total 60 observations.

Figure 5: Time-series plot of monthly jump and total betas



Note: The dotted and solid lines represent monthly jump and total betas from January 2012 to December 2016, respectively.

Table 6: The degree of fractional integration for monthly jump and total beta and covariation

	EA	TE	BA	RE	Market	Mean	Median
Jump beta	0.13	0.33	0.27	0.08	—	0.20	0.20
Jump covariation	0.31	0.31	0.26	0.02	0.31	0.24	0.31
Total beta	0.44	0.45	0.46	0.36	—	0.43	0.44
Total covariation	0.35	0.36	0.36	0.36	0.37	0.36	0.36

Note: This table reports the estimates of long-memory parameter d in $ARFIMA(0, d, 0)$ model for jump beta (second row), market jump variation and covariation for sector portfolios (third row), and the total ones (fourth and fifth rows), respectively. The last two columns report the average and median of the estimated d in each row.

considering the jump risk component in the market beta is meaningful to capture the statistical difference between the two betas.

4. Conclusion

Several recent studies investigate the fact that the premia associated with jump risks sometimes appears to be different from the premia associated with continuous risks. This study measures market jump risk factor loading determined by the covariance between market jump and contemporaneous asset high-frequency returns. The estimates show that annually aggregated jump beta is smooth over time, while monthly aggregated jump beta captures large changes in the sensitivity of sector portfolios to the market jumps. This study also investigates the time-series properties of realized jump beta, such as the constancy over fixed intervals of time and the degree of fractional integration. We have new evidence in the Japanese financial market that in contrast to monthly aggregated jump beta, the constancy of annual jump beta is always rejected, and the time-varying monthly jump beta has a smaller degree of fractional integration and less persistence compared with the total realized beta largely including the continuous risk component of asset returns. We note that the empirical analysis in this study is limited to the statistical tests and estimation of time-series models for jump beta. Therefore, it would be worthwhile to investigate whether or not separating the continuous and discontinuous betas also has important implications for practical portfolio and risk management.

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