Hyperbolic Discounting and State-Dependent Commitment

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Hyperbolic Discounting and State-Dependent Commitment*

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Abstract

In a stochastic economy with uninsurable endowment risk, we establish the condition under which hyperbolic-discounting consumers commit to a future consumption path by utilizing both illiquid capital and borrowing constraints as a commitment device. There is always the possibility that a state-dependent commitment can be adopted as an equilibrium consumption strategy. On a path leading to low future endowment, the current self can commit to their own optimal consumption path, which is undesirable for future selves. In contrast, along the path with a high future endowment, the current self cannot make a commitment and must accept a consumption allocation that future selves will revise. We also examine the effect of financial development on economic growth through consumption commitment. Relaxation of borrowing constraints across stages of life changes the current self's incentive to control future selves' behavior in different ways, and thus may or may not promote illiquid capital accumulation.

Keywords: Borrowing constraint, Commitment, Endowment risk, Hyperbolic discounting, Illiquidity.

JEL Classification Numbers: E21, E70, G12, G51.

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1 Introduction

People often face an internal struggle between the present self and future selves. One cause of such a struggle is a change in preference, which can be modeled by, for example, hyperbolic discounting (Frederick et al. 2002). Numerous studies, such as Strotz (1956), Phelps and Pollak (1968), and Laibson (1997), investigate decision making under time-inconsistent preferences. However, most of them focus mainly on intertemporal consumption choices, and scarcely treat commitment decisions across stochastic states of nature. This study contributes to the literature by analyzing the commitment decision of hyperbolic-discounting individuals in a risky environment and showing that the commitment can be state dependent—people may or may not make a consumption commitment according to states that will arise in the future. On the path along which a state with a low level of income will be realized, the current self is able to make a commitment that future selves will follow, even though it is desirable for the current young self but not for future selves. In contrast, along the path with a high level of future income, the current self is incapable of committing to their own optimal consumption allocation, and future selves reoptimize flexibly depending on the realized level of income. In other words, there may be a situation where even rational consumers do not fully utilize commitment devices. We also show that financial development changes the power balance between the current self and future selves when they decide consumption plans and affects economic growth as well as commitment decisions.

A large body of literature explores decision making of agents with time-inconsistent preferences and emphasizes the importance of utilizing commitment devices. The idea of commitment goes back at least to Strotz (1955), who demonstrates that commitment is useful for avoiding time-inconsistent decision making. With a hyperbolic-discounting preference, Laibson (1997) finds that a combination of illiquid assets and borrowing constraints plays a significant role as a commitment device in an economy with no risk factor. These seminal works have led to an expanding literature on hyperbolic discounting and commitment (see Bryan et al. 2010 for a survey on commitment). For example, Angeletos et al. (2001) examine a quantitative implication of Laibson (1997). Compulsory savings through social security systems (Schwarz and Sheshinski 2007) and introduction of saving floors (Malin 2008) are also shown to work as commitment devices. Despite these theoretical predictions, experimental studies often report that only a small fraction of subjects voluntarily utilizes commitment devices (Giné et al. 2010; Augenblick et al. 2015). Laibson (2015) stresses the psychological or monetary costs involved in making a commitment as one possible reason. We point out another: the presence of income risk hinders individuals from making a commitment. Concretely, we provide the full characterization of equilibrium consumption strategies in a risky situation and show the possibility that individuals cannot make a commitment, particularly in a highly volatile economy.

Amador et al. (2006) is a notable exception treating the consumption decision of hyperbolic-discounting individuals in a stochastic economy.[1] In the presence of taste shocks consisting of private information on future selves, the current self encounters a conflicting choice between commitment and flexibility—namely, either removing all ex-post choices from future selves or leaving flexible options open to them. Taking into account an incentive compatibility condition for future selves, the current self makes a take-it-or-leave-it offer to future selves. When the time-inconsistency in preference is slight, the current self can compel future selves to accept the current self's optimal consumption plan. However, as the time-inconsistency strengthens, the current self cannot help incorporating the future selves' tastes into a consumption plan so as to make future selves reveal their true preferences. While Amador et al. clarify how incentive-compatible consumption plans relate to the degree of hyperbolic discounting, we explicitly introduce borrowing constraints and illiquid capital into a stochastic dynamic optimization model as a commitment device. This enables us to associate the feasibility conditions of commitment with the degree of borrowing constraints and to examine the effect of financial development on economic growth through the accumulation of illiquid capital.

The relationship between borrowing constraints and economic growth is an open issue that attracts a lot of interest.[2] For instance, as demonstrated by Jappelli and Pagano (1994, 1999), relaxing borrowing limits leads to encouragement of consumption and reduction of capital accumulation, with long-term negative effects. In a general equilibrium model with lenders and constrained borrowers, Biederman (2000) finds that a relaxed constraint on borrowing has a positive or negative welfare effect. Constantinides et al. (2002) indicate that the presence of borrowing constraints forces a decrease in demand for risky assets, which explains the empirical fact of an abnormally high equity premium. Kiyotaki and Moore (1997) build a model in which the borrowing constraint on productive investors causes inefficient credit allocation and accounts for a prolonged economic slump through degradation of collateral values.[3] A strand of research analyzes the growth effect of hyperbolic discounting. A crucial conclusion is that the neoclassical growth model has a similar structure between standard exponential and hyperbolic discounting (Barro 1999; Krusell et al. 2002; Krusell and Smith 2003; Strulik 2015). However, no such study takes into account commitment devices. In this paper, we provide a new insight by paying attention to the role of borrowing constraints and illiquid capital as a commitment device.

In our model, a consumer encounters different borrowing constraints in different stages of life. With the borrowing constraint relaxed, the current self gains stronger commitment power: they can accumulate more illiquid physical capital, thereby controlling the behavior of future selves. This enhances the welfare of the current self but worsens that of future selves. On the other hand, as the borrowing constraints fu-

[1] Ambrus and Egorov (2013) extensively reexamine the results of Amador et al. (2006).

[2] Aiyagari (1994) and Huggett (1997) investigate the property of a neoclassical growth model with uninsured idiosyncratic income risk and borrowing constraints. See Townsend (2010) and Lerner and Tufano (2011) for surveys regarding financial innovations, economic growth, and welfare.

[3] See Gregorio (1996) and Kitaura (2012) for a model incorporating human capital accumulation.

ture selves encounter are relaxed, it becomes difficult for the current self to control the consumption decisions of future selves, therefore bringing less incentive to the current self to accumulate illiquid capital. This is harmful to the current self but beneficial to future selves. Thus, the total effect of financial development on economic growth and welfare is generally ambiguous. In contrast to previous studies, with time-consistent preferences, our results reflect the current self's incentive to manage time-inconsistent behavior through accumulating illiquid capital.

The rest of the paper proceeds as follows. Section 2 presents the basic structure of the model. Section 3 solves the optimization problem of a consumer who has time-inconsistent multiple selves inside. Section 4 compares welfare among some consumption strategies elaborated by the current self and determines a subgame-perfect equilibrium in a game played by multiple selves. Section 5 discusses the effect of financial development on capital accumulation and welfare. Section 6 concludes the paper.

2 The Model

Using an overlapping-generations model with uninsured endowment risk, we study the condition under which hyperbolic-discounting consumers can commit to a future consumption path with the help of illiquid assets and borrowing constraints. Individuals live in one of three periods: young, middle-aged, and old. The population size of each generation is normalized to unity.

The generation born in period t receives a non-stochastic endowment, $e_t^y(>0)$, when young and a stochastic endowment, $e_{t+1}^{mi}(>0)$, when middle-aged:

$$e_{t+1}^{mi} = \begin{cases} e_{t+1}^{mb} & \text{with probability} \;\; \pi, \\ e_{t+1}^{mg} & \text{with probability} \;\; 1-\pi, \end{cases}$$

where $e_{t+1}^{mb} < e_{t+1}^{mg}$ and $0 < \pi < 1$. The risk on endowment is uninsurable.

Being endowed with e_t^y, the young take out short-term loan b_{t+1}^y, which is repaid in the next period, and invest in illiquid physical capital $z_{t+2}^y(\geq 0)$, which yields Az_{t+2}^y units of output two periods later and cannot be liquidated before maturity. For analytical simplicity, we omit the young's consumption. The flow budget equation when young is then given by

$$z_{t+2}^y = e_t^y + b_{t+1}^y. \tag{1}$$

Given the gross loan rate of interest r and the endowment realized at the beginning of period $t+1$, the middle-aged repay the existing loan and roll over loans for consumption c_{t+1}^{mi}:

$$c_{t+1}^{mi} + rb_{t+1}^y = e_{t+1}^{mi} + b_{t+2}^{mi} \qquad \text{for} \;\; i = g, b, \tag{2}$$

where b_{t+2}^{mi} denotes the amount of new loans. The old then obtain the return on physical capital investment and consume the rest after repaying the loan:

$$c_{t+2}^{oi} + rb_{t+2}^{mi} = Az_{t+2}^y \qquad \text{for} \;\; i = g, b, \tag{3}$$

where c^{oi}_{t+2} represents consumption when old. b^{mi}_{t+2}, c^{mi}_{t+1}, and c^{oi}_{t+2} are state-dependent in the absence of insurance markets.

Suppose a small open economy in which the loan interest rate r is constant over time and individuals are indifferent between lending in the international loan market and investing physical capital by themselves:

Assumption 1: $r^2 = A.$

Following Phelps and Pollak (1968), Pollak (1968), and Laibson (1997), we model a hyperbolic-discounting consumer as a sequence of different *selves* with different preferences. The expected utility for the young *self* is

$$\pi\left(\ln c^{mb}_{t+1} + \delta \ln c^{ob}_{t+2}\right) + (1-\pi)\left(\ln c^{mg}_{t+1} + \delta \ln c^{og}_{t+2}\right), \tag{4}$$

where $0 < \delta < 1$. As time passes, the discount factor declines, and an individual becomes more impatient to consume now rather than in the future. The utility for the middle-aged *self* is then characterized by

$$\ln c^{mi}_{t+1} + \beta\delta \ln c^{oi}_{t+2} \qquad \text{with } \ 0 < \beta < 1. \tag{5}$$

Here, β measures the degree of time inconsistency in preference. The optimal consumption path for the middle-aged self deviates from that for the young self. As pointed out by Laibson (1997), however, the young manage to control the middle-aged's behavior by making use of illiquid assets and borrowing constraints.

Due to imperfect information in the loan market, lenders are assumed to require sufficient collateral. As a result, the maximum amount the young can borrow is constrained at a fraction $\phi^y (> 0)$ of the present value of expected income obtained in the next period:

$$b^y_{t+1} \leq \phi^y \frac{\mathrm{E}_t[e^{mi}_{t+1}]}{r} \qquad \text{with } \ 0 \leq \phi^y < \frac{e^{mb}_{t+1}}{\mathrm{E}_t[e^{mi}_{t+1}]}.$$

The second inequality in the second equation ensures that the repayment rb^y_{t+1} is less than the flow income e^{mi}_{t+1} even if state b arises. Using (1), we can rewrite the borrowing constraint as

$$z^y_{t+2} \leq \bar{z}_{t+2} \equiv e^y_t + \phi^y \frac{\mathrm{E}_t[e^{mi}_{t+1}]}{r}, \tag{6}$$

where $\bar{z}_{t+2}$ is an upper bound of physical capital investment consisting of flow income e^y_t and the borrowing $\phi^y \mathrm{E}_t[e^{mi}_{t+1}]/r$ when young. Similarly, the borrowing constraint the middle-aged face is

$$b^{mi}_{t+2} \leq \phi^m \frac{A z^y_{t+2}}{r} = \phi^m r z^y_{t+2} \qquad \text{with } \ 0 \leq \phi^m < 1, \tag{7}$$

where the equality in the first equation comes from assumption 1.

3 Optimization

We consider a subgame-perfect equilibrium in a game played among multiple selves. A consumer is said to be sophisticated if the young self elaborates a time-consistent consumption plan that accurately predicts the reaction of the middle-aged self. We first derive the middle-aged self's reaction function and then solve the utility maximization problem for the sophisticated young self. (In what follows, we abbreviate the time subscript to simplify exposition.)

3.1 The middle-aged self

Let us analyze the middle-aged's behavior. After the endowment shock is realized, the middle-aged who are in state $i = b, g$ maximize their own utility (5) subject to the budget equations (1)–(3) and the borrowing constraint (7):

$$\max_{b^{mi}} \ \ln\left[e^{mi} + b^{mi} - r(z^y - e^y)\right] + \beta\delta \ln(Az^y - rb^{mi}) + \mu^{mi}(\phi^m r z^y - b^{mi}),$$

where μ^{mi} is a Lagrange multiplier for (7). The first-order necessary conditions are

$$\frac{1}{e^{mi} + b^{mi} - r(z^y - e^y)} - \frac{\beta\delta r}{Az^y - rb^{mi}} - \mu^{mi} = 0,$$
$$\mu^{mi}(\phi^m r z^y - b^{mi}) = 0, \quad \mu^{mi} \geq 0, \quad b^{mi} \leq \phi^m r z^y.$$

Two cases may arise, depending on whether the borrowing constraint is binding: either $b^{mi} = \phi^m r z^y$ or $b^{mi} < \phi^m r z^y$.

In the case of $b^{mi} < \phi^m r z^y$, we obtain $\mu^{mi} = 0$ and

$$b^{mi} = rz^y - \frac{\beta\delta}{1+\beta\delta} rW^i, \qquad c^{mi} = \frac{rW^i}{1+\beta\delta}, \qquad c^{oi} = \frac{\beta\delta A W^i}{1+\beta\delta}, \tag{8}$$

where W^i represents the present value of lifetime income when state i arises:

$$W^i \equiv e^y + \frac{e^{mi}}{r}, \qquad \text{where } \ W^b < W^g.$$

The consumption allocation (8), which is optimal for the middle-aged, is achievable as long as the middle-aged's borrowing constraint is not strictly binding:

$$\textbf{Condition 1a:} \quad b^{mi} < \phi^m r z^y \quad \Leftrightarrow \quad 0 \leq z^y < \frac{\beta\delta W^i}{(1-\phi^m)(1+\beta\delta)}.$$

The other case is the one where the borrowing amount reaches the ceiling:

$$\mu^{mi} \geq 0, \qquad b^{mi} = \phi^m r z^y,$$

which determines the consumption allocation as

$$c^{mi} = r\left[W^i - (1-\phi^m)z^y\right], \qquad c^{oi} = (1-\phi^m)Az^y. \tag{9}$$

This implies that the middle-aged cannot smooth consumption intertemporally:

$$\textbf{Condition 1b:} \quad \frac{1}{c^{mi}} \geq \frac{\beta\delta r}{c^{oi}} \quad \Leftrightarrow \quad \frac{\beta\delta W^i}{(1-\phi^m)(1+\beta\delta)} \leq z^y.$$

The presence of borrowing ceilings prevents a consumer from increasing consumption when middle-aged. While this is harmful to the impatient middle-aged, it is beneficial to the patient young. By adjusting the level of illiquid physical capital, the young may actually avoid this unpleasant situation for themselves.

Keeping in mind that conditions 1a and 1b depend on the realized state $i = b, g$, we can summarize the results in the following lemma:

Lemma 1 *The middle-aged's reaction is a function of illiquid capital left by the young.*

(a) **Full flexibility***: If* $0 \leq z^y < \frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)}$*, then the middle-aged's borrowing constraint is not binding in both states* b *and* g *and consumption is given by (8).*

(b) **State-dependent commitment***: If* $\frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)} \leq z^y < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$*, then the middle-aged's borrowing constraint is binding only in state* b*. Consumption in state* g *is given by (8), in which* $i = g$*, whereas consumption in state* b *is given by (9), in which* $i = b$*.*

(c) **Full commitment***: If* $\frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq z^y$*, then the middle-aged's borrowing constraint is binding in both states* b *and* g *and consumption is given by (9).*

A smaller z^y brings lower income when old, so that the middle-aged have less incentive to borrow and can flexibly choose their own consumption allocation, not being restricted by the borrowing limit. This case is called full flexibility. However, as z^y increases, so does the incentive to borrow, and the amount of borrowing eventually reaches the limit. The situation occurs first in state b, where the flow income when middle-aged, e^{mi}, is lower. The middle-aged self is deprived of a flexible choice, whereas the young self can commit to the consumption path they desire. Hence, the decision on whether to commit depends on the realized state. We call this type of consumption schedule state-dependent commitment. With a sufficiently large z^y, the young can fully commit to their own optimal consumption allocation in both states b and g, whereas the middle-aged have no option but to follow the young's decision. This case is called full commitment.

3.2 The young self

We next derive the level of illiquid capital, which determines an equilibrium consumption path from lemma 1. Obviously, the young want to commit to the path that maximizes their own utility by sufficiently accumulating capital. However, we show that they may or may not be able to afford to do so.

3.2.1 Full commitment

In the case of full commitment presented in lemma 1c, the young can control the middle-aged's behavior in both states of nature so as to maximize the young's utility (4) subject to the young's borrowing constraint (6) and the middle-aged's reaction function (9):

$$\begin{aligned}\max_{z^y} \quad & \pi \ln r \left[W^b - (1-\phi^m)z^y\right] + (1-\pi)\ln r\left[W^g - (1-\phi^m)z^y\right] \\ & + \delta \ln(1-\phi^m)Az^y + \mu^y(\bar{z} - z^y),\end{aligned}$$

where μ^y is a Lagrange multiplier for (6). The first-order necessary conditions are

$$-\left[\frac{\pi(1-\phi^m)}{W^b-(1-\phi^m)z^y} + \frac{(1-\pi)(1-\phi^m)}{W^g-(1-\phi^m)z^y}\right] + \frac{\delta}{z^y} - \mu^y = 0, \tag{10}$$

$$\mu^y(\bar{z}-z^y) = 0, \quad \mu^y \geq 0, \quad z^y \leq \bar{z}. \tag{11}$$

In the case of $z^y < \bar{z}$ or $\mu^y = 0$, the level of capital is calculated as [4]

$$\begin{aligned} z^{FC} \equiv & \frac{1}{2(1-\phi^m)(1+\delta)}\Big\{(1-\pi+\delta)W^b + (\pi+\delta)W^g \\ & - \sqrt{[(1-\pi+\delta)W^b + (\pi+\delta)W^g]^2 - 4\delta(1+\delta)W^bW^g}\Big\}.\end{aligned} \tag{12}$$

This provides the following Euler equation:

$$\frac{1-\pi}{c^{mg}} + \frac{\pi}{c^{mb}} = \frac{\delta r}{c^{oi}} \qquad \text{for} \quad i = g, b.$$

From the viewpoint of the young self, they can smooth consumption over time and across states of nature and also has commitment power in both state b and state g. Full commitment is thus desirable for the young self but not for the middle-aged self. From the viewpoint of the middle-aged self, they have no flexible option for diversifying the endowment risk intertemporally—in fact, consumption when old is the same across states, at $c^{ob} = c^{og} = (1-\phi^m)Az^{FC}$, while consumption c^{mi} is volatile in response to the realized level of endowment. This strategy is feasible as long as the middle-aged's borrowing constraint is binding in both states of nature and the young's is not:

$$\textbf{Condition 2a:} \quad \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq z^{FC} < \bar{z},$$

where the first equality is from lemma 1c and the second equality means that the young's borrowing constraint is not binding.

Let us now turn to the case of $z^y = \bar{z}$ or $\mu^y \geq 0$. The borrowing ceiling hampers the young's intertemporal smoothing of consumption, although commitment remains possible in both states of nature:

$$z^y = \bar{z}, \qquad \frac{1-\pi}{c^{mg}} + \frac{\pi}{c^{mb}} \leq \frac{\delta r}{c^{oi}}.$$

[4] See appendix A for the derivation.

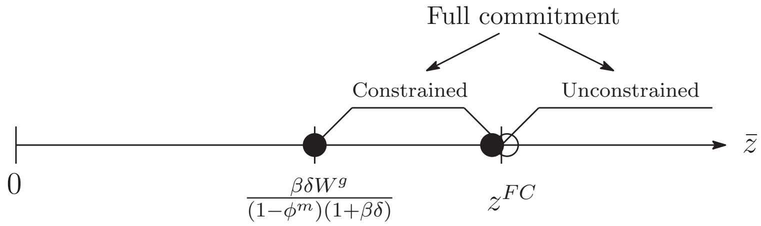

Figure 1: Full commitment.

While both the young and the middle-aged face each borrowing ceiling, this situation has a different meaning between the two groups. As seen in the second equation above, for hyperbolic-discounting individuals, the patient young self desires to shift income to the third period of life; in contrast, the impatient middle-aged self wants to decrease the old's consumption, because condition 1b holds now:

$$\frac{1}{c^{mi}} > \frac{\beta\delta r}{c^{oi}} \quad \text{for} \quad i = g, b.$$

These conditions are equivalent to

$$\textbf{Condition 2b:} \quad \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq \bar{z} \leq z^{FC},$$

where the first inequality comes from substituting $z^y = \bar{z}$ into the condition in lemma 1c and the second inequality implies the binding borrowing constraint of the young.

The results are summarized in the following lemma and illustrated in figure 1:

Lemma 2 (Full commitment) *If* $\frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq z^{FC}$ *for the* z^{FC} *in (12), the young can make a full commitment by which they can commit to the consumption path in both states of nature. Moreover,*

(a) **Unconstrained***: Under condition 2a, the young are unconstrained by the borrowing limit in the first stage of life and can smooth consumption over time and across states of nature.*

(b) **Constrained***: Under condition 2b, the young are constrained by the borrowing limit in the first stage of life and are restricted to smoothing consumption intertemporally.*

3.2.2 State-dependent commitment

Lemma 2 implies that there are some regions where full commitment is infeasible. Then, as in the case of Lemma 1b, the young reluctantly devise state-dependent commitment—that is, they make a commitment in state b, but they allow the middle-aged to have a flexible choice in state g.

In state g, the young have no choice but to follow the impatient middle-aged's decision, although they are willing to maximize their own utility. From (8), the consumption allocation in state g satisfies the following Euler equation:

$$\frac{1}{c^{mg}} = \frac{\beta\delta r}{c^{og}}.$$

By contrast, the consumption decision in state b is in the patient young's hand. The young choose the z^y that maximizes their own utility (4) subject to their borrowing constraint (6) and the middle-aged's reaction function (9), in which $i = b$:

$$\max_{z^y} \ \pi \left\{ \ln r \left[W^b - (1 - \phi^m) z^y \right] + \delta \ln(1 - \phi^m) A z^y \right\} + \mu^y (\bar{z} - z^y).$$

The first-order necessary conditions are (11) and

$$-\pi \left[\frac{1 - \phi^m}{W^b - (1 - \phi^m) z^y} - \frac{\delta}{z^y} \right] - \mu^y = 0.$$

In the case of $z^y < \bar{z}$ or $\mu^y = 0$, the level of capital is obtained by

$$z^{SDC} \equiv \frac{\delta W^b}{(1 - \phi^m)(1 + \delta)} \left(< z^{FC} \right), \tag{13}$$

which is smaller than the z^{FC} in (12) and gives the Euler equation,

$$\frac{1}{c^{mb}_{t+1}} = \frac{\delta r}{c^{ob}_{t+2}}.$$

In comparison with full commitment, the young here cannot diversify the endowment risk across states of nature. In addition, the discount factor in the Euler equation differs across states—it is $\beta\delta$ in state g and δ in state b. This strategy is possible if

$$\textbf{Condition 3a:} \quad z^{SDC} < \frac{\beta\delta W^g}{(1 - \phi^m)(1 + \beta\delta)}, \quad z^{SDC} < \bar{z}.$$

The first condition means that the middle-aged's borrowing constraint is binding in state b but not in state g. We obtain it by substituting the z^{SDC} in (13) into the condition in lemma 1b and noting that $\frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)} < z^{SDC}$ is always fulfilled with $\beta < 1$. The second condition is the non-binding borrowing constraint of the young.

In the case of $z^y = \bar{z}$ or $\mu^y \geq 0$, the presence of a borrowing ceiling disturbs the intertemporal consumption smoothing of the young in state b:

$$\frac{1}{c^{mb}} \leq \frac{\delta r}{c^{oi}}.$$

This is the case where the young's borrowing amount reaches the ceiling and, simultaneously, the condition in lemma 1b is satisfied. In this case, either of the following two conditions is required:

$$\textbf{Condition 3b:} \quad \frac{\beta\delta W^b}{(1 - \phi^m)(1 + \beta\delta)} \leq \bar{z} \leq z^{SDC} < \frac{\beta\delta W^g}{(1 - \phi^m)(1 + \beta\delta)},$$

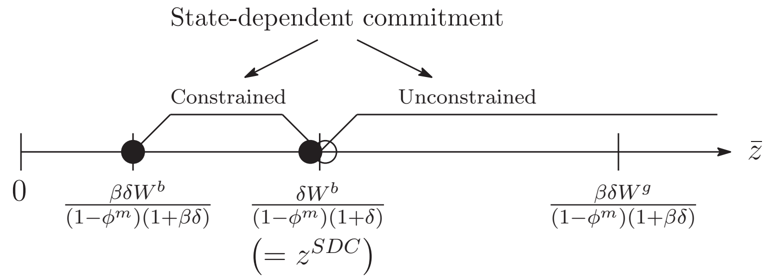

Figure 2a: State-dependent commitment for $z^{SDC} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$.

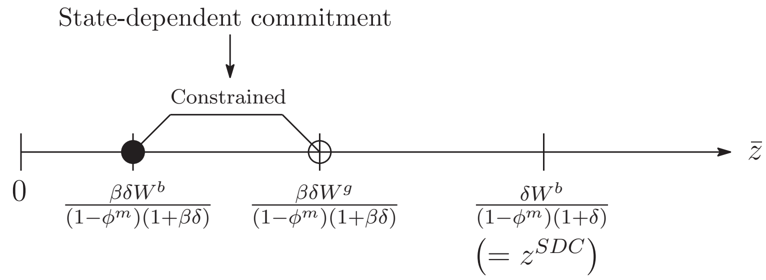

Figure 2b: State-dependent commitment for $\frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq z^{SDC}$.

$$\textbf{Condition 3c:} \quad \frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)} \leq \bar{z} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq z^{SDC}.$$

Therefore, we conclude that:

Lemma 3 (State-dependent commitment) *The young can make a state-dependent commitment by which they can commit to the consumption path in state b but not in state g, and furthermore:*

(a) **Unconstrained***: Under condition 3a, the young are unconstrained by the borrowing limit in the second stage of life and can smooth consumption over time in state b.*

(b) **Constrained***: Under either condition 3b or condition 3c, the young are constrained by the borrowing limit in the second stage of life and restricted to smoothing consumption intertemporally in state b.*

Figure 2a describes the case where $z^{SDC} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$, whereas figure 2b illustrates that for $\frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq z^{SDC}$.

3.2.3 Full flexibility

In the region where both full and state-dependent commitment are impossible, the young are governed by the middle-aged's decisions in both states of nature. In other words, the middle-aged have a fully flexible option in every situation. The consumption allocation is thus determined by (8), satisfying the following Euler equations with the lower discount factor, $\beta\delta$:

$$\frac{1}{c^{mi}} = \frac{\beta\delta r}{c^{oi}} \qquad \text{for} \quad i = g, b.$$

This case is possible for any z^y and $\bar{z}$ such that

$$\textbf{Condition 4:} \quad 0 \leq z^y \leq \bar{z} < z^{FF},$$

$$\text{where} \quad z^{FF} \equiv \frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)} (< z^{SDC} < z^{FC}). \tag{14}$$

Here, z^{FF} is smaller than the z^{FC} in (12) as well as the z^{SDC} in (13). This condition comes from the young's borrowing constraint (6) and the condition in lemma 1a. The amount of capital investment is indeterminate within the range of condition 4 but it does not affect the consumption allocation the middle-aged will decide on.[5] It is formally noted that:

Lemma 4 (Full flexibility) *As long as the level of capital is under that in condition 4, the young cannot help taking the full-flexibility strategy by which the consumption allocation in both states of nature is flexibly determined by the middle-aged.*

4 Welfare and Equilibrium

As shown in the previous section, there are potentially three kinds of consumption strategies relevant here: full commitment, state-dependent commitment, and full flexibility. When the region within which each strategy is feasible overlaps, the young adopt a welfare-maximizing strategy, and a subgame-perfect equilibrium is determined.

Compare the welfare attained by each strategy in the relevant region. It is intuitive that:

Lemma 5 (Welfare comparison) *In terms of the young's welfare, full commitment is superior and full flexibility is inferior to the other two strategies regardless of whether the young's borrowing constraint is binding.*

Proof: See appendix B.

It is straightforward that full commitment (full flexibility) is the best (worst) strategy considering the young self's utility. Furthermore, this lemma states that this welfare ranking holds even when the young's intertemporal consumption choices are constrained by borrowing limits.

[5]The indeterminacy of equilibrium capital is due to the linear production function and free international lending and borrowing. Mundell (1957) points out that international specialization patterns of production become indeterminate in the presence of international capital movement. Ono and Shibata (2010) show that this property also holds in a dynamic-optimization setting.

Given the welfare ranking in this lemma, cases are divided into three types according to the degree of time-inconsistency caused by hyperbolic discounting. Figure 3 classifies the strategy adopted in each case and in each region. The horizontal axis $\bar{z}$ stands for the maximum amount the young can invest in capital.

Note that the z^{FC} in (12) is independent of β. Figure 3a illustrates the case with a lower degree of time-inconsistency in preference, that is, a β close to unity:[6]

$$\textbf{Condition 5a:} \quad \left(\frac{\delta W^b}{(1-\phi^m)(1+\delta)} <\right) z^{FC} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}.$$

In the stochastic environment, there is a possibility that the full-commitment equilibrium disappears and the state-dependent commitment is instead employed. Being endowed with higher income in state g, W_t^g, as well as the larger β (condition 5a), the middle-aged have less incentive to borrow and are beyond the control of the young. In contrast, if the income in state b, W_t^b, is low, or if the young are capable of sufficiently investing in illiquid capital due to the higher borrowing capacity $\bar{z}$, then the young can commit to the desired consumption path in state b (see the region where $z^{FF} \le \bar{z}$ in figure 3a). What is worse, if the higher income is also received in state b or if the borrowing limit the young face is severe, the middle-aged's behavior is completely out of control, and full flexibility is a unique equilibrium strategy (see the region where $0 \le \bar{z} < z^{FF}$ in figure 3a). This establishes the following proposition:

Proposition 1 *Under condition 5a, there is no full-commitment equilibrium; instead, the state-dependent commitment is an equilibrium strategy under $z^{FF} \le \bar{z}$ as well as condition 5a.*

Figure 3b considers a higher degree of time-inconsistency measured by a lower β:

$$\textbf{Condition 5b:} \quad \frac{\delta W^b}{(1-\phi^m)(1+\delta)} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \le z^{FC}.$$

Every type of equilibrium, including the full-commitment strategy, emerges. The lower β raises the borrowing demand of the impatient middle-aged and makes it easier for the young to adopt the full-commitment strategy.

Figure 3c treats a significantly higher degree of time-inconsistency, such that

$$\textbf{Condition 5c:} \quad \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \le \frac{\delta W^b}{(1-\phi^m)(1+\delta)} \left(< z^{FC}\right),$$

The lower β needs more illiquid assets to control the middle-aged and eliminates the region for state-dependent commitment unconstrained by the young's borrowing ceiling. As β approaches zero, full commitment is a unique equilibrium strategy even if the young's borrowing constraint is extremely tight, that is, $\bar{z} \to 0$.

The conditions for state-dependent commitment to arise, presented in figure 3, are included in:

[6] Remember that applying (12) and (13) to lemma 1 requires $z^{SDC} \equiv \frac{\delta W^b}{(1-\phi^m)(1+\delta)} < z^{FC}$.

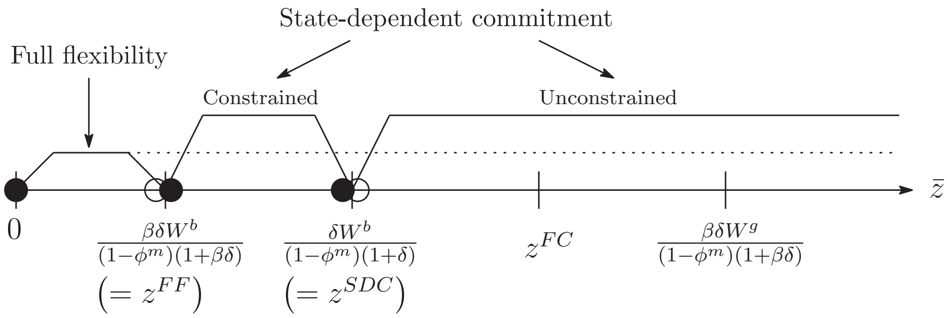

Figure 3a: $z^{FC} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$ (condition 5a).

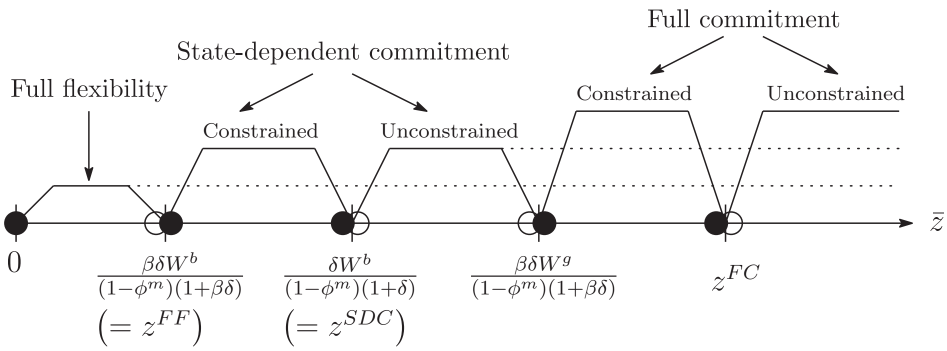

Figure 3b: $\frac{\delta W^b}{(1-\phi^m)(1+\delta)} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq z^{FC}$ (condition 5b).

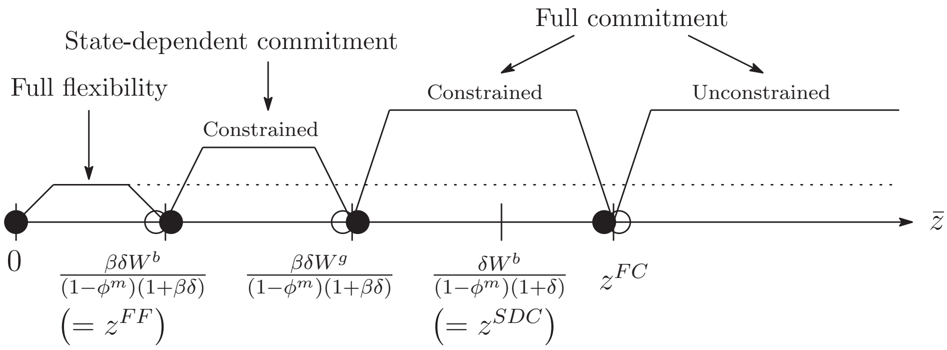

Figure 3c: $\frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq \frac{\delta W^b}{(1-\phi^m)(1+\delta)}$ (condition 5c).

Proposition 2 *State-dependent commitment is an equilibrium strategy if any of the following conditions are fulfilled:*

(a) condition 5a and $\frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)} \leq \bar{z}$.

(b) condition 5b and $\frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)} \leq \bar{z} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$.

(c) condition 5c and $\frac{\beta\delta W^b}{(1-\phi^m)(1+\beta\delta)} \leq \bar{z} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$.

Keeping the average income $\mathrm{E}[e^{mi}]$ constant, an increased income volatility measured by the difference $W^g - W^b$ raises the income in state g but decreases it in state b. This widens the region within which state-dependent commitment is adopted; in other words, the presence of income risk prevents the current self from making a commitment for all possible states in the future. The result is consistent with the findings of experimental studies reporting that the voluntary demand for commitment devices is much smaller than that estimated in theoretical predictions (Giné et al. 2010; Augenblick et al. 2015; Laibson 2015). Even if individuals have access to commitment devices, they cannot effectively utilize them to accomplish full commitment; furthermore, there are some stochastic situations where commitment is impossible in some states of nature (propositions 1 and 2).

5 Financial Development

This section investigates an impact of financial development on consumption decisions and capital accumulation.

5.1 Insurance markets

The purpose of this study is to clarify the commitment strategy employed in a risky situation. What would the consumption decision be if there were no endowment risk, or equally, if an insurance market were complete? If the risk were perfectly diversified either within or among countries, it would hold that:

$$W^b = W^g = \mathrm{E}[W^i], \qquad z^{FC} = z^{SDC} = \frac{\delta \mathrm{E}[W^i]}{(1-\phi^m)(1+\delta)},$$

from (12) and (13). Neither condition 5a nor 5b is fulfilled, but condition 5c remains, thereby transforming figure 3c to figure 4. Instead of causing the state-dependent commitment equilibrium to vanish, this leads the range of full flexibility and full commitment to widen.

Corollary 1 *An economy with complete insurance markets has a two-alternative strategy: full commitment is an equilibrium strategy if* $\bar{z} \geq z^{FF}$*; otherwise, full flexibility is an equilibrium strategy.*

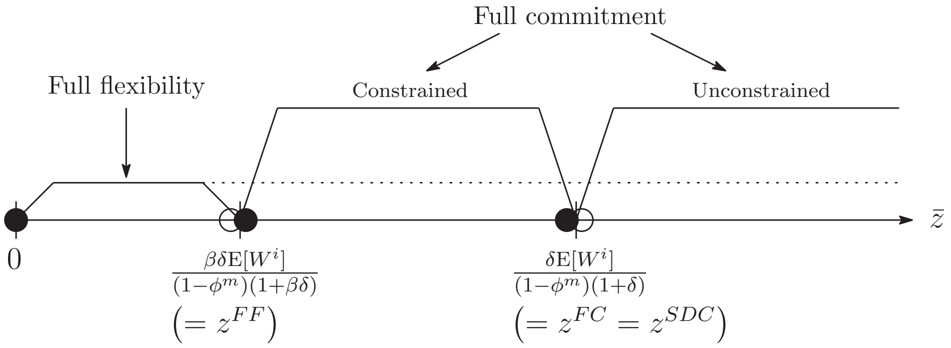

Figure 4: A non-stochastic economy.

Laibson (1997) emphasizes an important role of combining illiquid assets and borrowing constraints as a commitment device in a deterministic world. He imposes assumption A1 in his paper (p. 452) to concentrate on the unconstrained full commitment equilibrium—the condition $z^{FC} = z^{SDC} < \bar{z}$ in our context. More precisely, he assumes an increasing path of flow income, which brings a strong borrowing motive in the current self and makes the borrowing constraint bind the future self. This study complements his contribution by taking account of not only income risk but also the entire parameter range and showing the possibility for state-dependent commitment to arise.

5.2 Borrowing constraints

It is controversial whether the financial innovation that relaxes borrowing limits encourages economic growth and enhances welfare (e.g., Jappelli and Pagano 1994, 1999; Biederman 2000). Our model contains the two borrowing constraints a consumer encounters in different stages of life, which affect capital accumulation in contrary directions.

The level of illiquid capital under condition 5b, for example, is illustrated in figure 5.[7] A rise in the maximum loan-to-value ratio for the young, ϕ^y, raises $\bar{z}$ and promotes capital accumulation z^y. It strengthens the young's commitment power, so that the young's welfare improves and the middle-aged's worsens. Conversely, an increase in the maximum loan-to-value ratio for the middle-aged, ϕ^m, raises the boundary conditions uniformly for a given $\bar{z}$ and discontinuously reduces capital z^y. This is because a relaxation of the middle-aged's borrowing constraint makes it difficult to adopt the commitment strategy for the young, and hence the young need less illiquid capital as a commitment device. Welfare is also influenced inversely.

Corollary 2 *A relaxation of borrowing constraints has an ambiguous effect on economic growth and welfare.*

[7] In figure 5, the shadowed area, $0 \leq \bar{z} < z^{FF}$, represents the indeterminate level of capital shown in lemma 4.

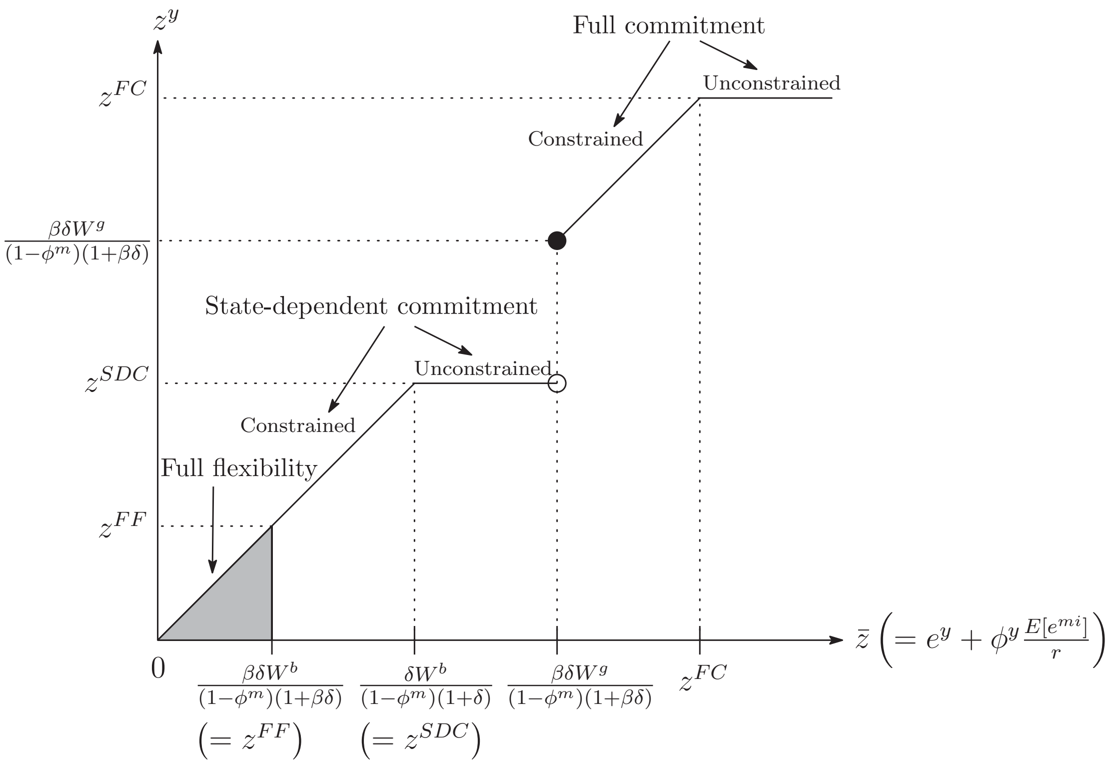

Figure 5: Borrowing constraints and illiquid capital.

(a) A rise in ϕ^y boosts capital accumulation. It is beneficial to the young self and harmful to the middle-aged self.

(b) A rise in ϕ^m depresses capital accumulation. It is harmful to the young self and beneficial to the middle-aged self.

The present relation between borrowing constraints and economic growth differs from that in the previous literature with time-consistent preferences. In an overlapping-generations model such as those by Jappelli and Pagano (1994, 1999) and Biederman (2000), the relaxation of borrowing constraints alters intergenerational income distribution through an increased or decreased savings motive in the existing generation and thus affects the welfare of future generations. However, in our model, internal struggle within a single person matters. The presence of borrowing constraints gives the young self an incentive to manage the behavior of the middle-aged self and changes the demand for illiquid capital as a commitment device. Thus, incompleteness in the financial market affects different selves in different ways and may or may not promote capital accumulation through an increased or decreased commitment motive in the young self.

6 Conclusion

Using an overlapping-generations model with endowment risk, we clarify how hyperbolic-discounting consumers utilize illiquid capital and borrowing constraints as a commitment device. There is always the possibility that the current self can commit to an optimal path only in some state of nature—which is called the state-dependent commitment. This occurs notably in a highly volatile economy. This result is a problem specific to a stochastic world, which is rarely treated in the literature. We also provide a detailed welfare comparison and full characterization of equilibrium strategies, including full commitment and full flexibility.

In the presence of hyperbolic-discounting individuals, the financial development that relaxes borrowing constraints has two opposite effects on capital accumulation. It directly induces capital formation by the patient current self; however, it simultaneously allows the impatient future self to make a flexible consumption decision and lowers the current self's motive for accumulating capital as a commitment device. In general, the total effect on economic growth and welfare remains ambiguous.

Recently, humanity has been exposed to undiversified risks such as the global financial crisis of 2007-2008 and coronavirus disease 2019. In times of increased risk, our model predicts that people are more likely to select state-dependent commitment—a hyperbolic-discounting individual executes a predetermined plan on the path toward economic contraction, but leaves a flexible choice for the expansionary path in the future.

This study provides some directions for future research. First, an extension of our model to a dynamic stochastic general equilibrium setting can generate implications on asset pricing, such as the term structure of interest rates and the excess returns of holding illiquid assets. Second, introduction of a heterogeneous degree of borrowing limits and time discounting can enrich the quantitative and qualitative results of the model. Third, taking costs of self-control into account may alter the welfare effects of various economic policies, as pointed out in, for example, Krusell et al. (2010) and Laibson (2015).

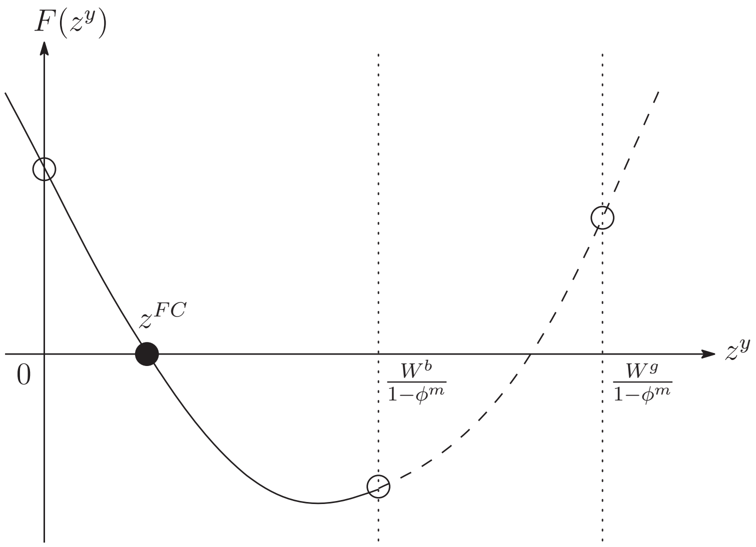

Figure A.1: The determination of z^{FC}.

Appendices

Appendix A: The derivation of (12)

This appendix derives (12) in the text. We can reduce (10) in which $\mu^y = 0$ to

$$F(z^y) \equiv (z^y)^2 - \frac{(1-\pi+\delta)W^b + (\pi+\delta)W^g}{(1-\phi^m)(1+\delta)} z^y + \frac{\delta W^b W^g}{(1-\phi^m)^2(1+\delta)} = 0,$$

which satisfies

$$F(0) = \frac{\delta W^b W^g}{(1-\phi^m)^2(1+\delta)} > 0,$$
$$F\left(\frac{W^b}{1-\phi^m}\right) = -\frac{\pi W^b (W^g - W^b)}{(1-\phi^m)^2(1+\delta)} < 0,$$
$$F\left(\frac{W^g}{1-\phi^m}\right) = \frac{(1-\pi) W^g (W^g - W^b)}{(1-\phi^m)^2(1+\delta)} > 0.$$

Figure A.1 depicts the shape of $F(z^y)$. Consumption in the full-commitment case is given by (9), and we find that:

$$c^{mi} \geq 0 \quad \text{if} \quad z^y \leq \frac{W^i}{1-\phi^m}.$$

Since we obtain $c^{mb} < 0$ for the larger root of $F(z^y) = 0$, z^y is uniquely determined by the smaller root, which is (12).

Appendix B: The proof of lemma 5

For analytical reasons, the level of capital in each state of nature is temporally denoted by z^i $(i = b, g)$. Using the consumption function (9), we can express the young's lifetime utility U as a function of z^b and z^g:

$$\begin{aligned} U &= U\left(z^b, z^g\right) \equiv \pi u^b + (1-\pi)u^g, \\ u^i &\equiv \ln r[W^i - (1-\phi^m)z^i] + \delta \ln(1-\phi^m)Az^i. \end{aligned}$$

Differentiating them with the level of capital yields

$$\begin{aligned} \frac{\mathrm{d}U}{\mathrm{d}z^y} &= \pi\frac{\mathrm{d}u^b}{\mathrm{d}z^b} + (1-\pi)\frac{\mathrm{d}u^g}{\mathrm{d}z^g}, \\ \frac{\mathrm{d}u^i}{\mathrm{d}z^i} &= \frac{(1-\phi^m)(1+\delta)}{[W^i-(1-\phi^m)z^i]z^i}\left[\frac{\delta W^i}{(1-\phi^m)(1+\delta)} - z^i\right]. \end{aligned} \tag{A.1}$$

Compare first the lifetime utility attained by each strategy when the young's borrowing is unconstrained. The maximum utility is achieved by full commitment at $z^b = z^g = z^{FC}$, which is in (12), and requires

$$\frac{\mathrm{d}U}{\mathrm{d}z^y} = 0, \qquad \frac{\mathrm{d}u^b}{\mathrm{d}z^b} < 0, \qquad \frac{\mathrm{d}u^g}{\mathrm{d}z^g} > 0,$$

where the first condition is equivalent to (10) with $\mu_t^y = 0$.

Utility under state-dependent commitment can be expressed by $z^b = z^{SDC}$ in (13) and $z^g = \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$, which provides consumption equal to (8) in state g:

$$\begin{aligned} c^{mg} &= r[W^g - (1-\phi^m)z^g] = \frac{rW^g}{1+\beta\delta}, \\ c^{og} &= (1-\phi^m)Az^g] = \frac{\beta\delta AW^g}{1+\beta\delta}. \end{aligned}$$

Evaluating (A.1) by these capital levels (or consumption levels) generates

$$\frac{\mathrm{d}U}{\mathrm{d}z^y} > 0, \qquad \frac{\mathrm{d}u^b}{\mathrm{d}z^b} = 0, \qquad \frac{\mathrm{d}u^g}{\mathrm{d}z^g} = \frac{(1-\phi^m)(1+\beta\delta)(1-\beta)}{\beta W^g} > 0.$$

Unless $\beta = 1$, the total marginal utility of capital is positive, reflecting the fact that the consumption allocation in state g is governed by the middle-aged and is not optimal for the young. The level of lifetime utility is hence shown to be lower than that of full commitment:

$$\underbrace{U\left(z^{SDC}, \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}\right)}_{\text{unconstrained state-dependent commitment}} < \underbrace{U\left(z^{FC}, z^{FC}\right)}_{\substack{\text{unconstrained} \\ \text{full commitment}}}. \tag{A.2}$$

The utility under the full-flexibility strategy corresponds to the case of $z^i = \frac{\beta\delta W^i}{(1-\phi^m)(1+\beta\delta)}$, satisfying

$$\frac{\mathrm{d}U}{\mathrm{d}z^y} > 0, \qquad \frac{\mathrm{d}u^i}{\mathrm{d}z^i} = \frac{(1-\phi^m)(1+\beta\delta)(1-\beta)}{\beta W^i} > 0.$$

As long as $\beta \neq 1$, the total marginal utility of capital is higher than that of state-dependent commitment because the young are under the control of the middle-aged in both states of nature. Therefore, the welfare ranking of these two strategies is [8]

$$\underbrace{U\left(z^{FF}, \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}\right)}_{\text{full flexibility}} < \underbrace{U\left(z^{SDC}, \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}\right)}_{\text{unconstrained state-dependent commitment}}. \tag{A.3}$$

By the same logic, we can evaluate the utility attained by state-dependent commitment and full commitment when hitting the young's borrowing limit. First, the consumption obtained under constrained state-dependent commitment equals that under $z^b = \bar{z}$ and $z^g = \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}$. From lemma 3, state-dependent commitment is feasible if $\bar{z}$ lies above the z^{FF} in (14). Thus, we have

$$\frac{\mathrm{d}u^b}{\mathrm{d}z^{FF}} \geq \frac{\mathrm{d}u^b}{\mathrm{d}\bar{z}} > 0, \qquad \frac{\mathrm{d}U}{\mathrm{d}z^{FF}} \geq \frac{\mathrm{d}U}{\mathrm{d}\bar{z}} > 0,$$

with equality when $\bar{z} = z^{FF}$. Consequently, even if the young's borrowing constraint is binding in the state-dependent commitment strategy, it is superior to the full-flexibility strategy in terms of the young's welfare:

$$\underbrace{U\left(z^{FF}, \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}\right)}_{\text{full flexibility}} \leq \underbrace{U\left(\bar{z}, \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}\right)}_{\text{constrained state-dependent commitment}}, \tag{A.4}$$

with equality when $\bar{z} = z^{FF}$.

Full commitment constrained by the borrowing ceiling exists if both condition 2b and condition 5b (or 5c) are fulfilled. As seen in figure 3b, conditions 2b and 5b equal

$$z^{SDC} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq \bar{z}.$$

This indicates a utility higher than that in unconstrained state-dependent commitment:

$$\underbrace{U\left(z^{SDC}, \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}\right)}_{\text{unconstrained state-dependent commitment}} < \underbrace{U\left(\bar{z}, \bar{z}\right)}_{\substack{\text{constrained}\\ \text{full commitment}}}. \tag{A.5}$$

[8] It is worth noting that for a time-consistent consumer with $\beta = 1$, the inequalities in (A.2) and (A.3) are not preserved and are replaced by equality, implying that the three strategies all ensure optimality.

In contrast, under conditions 2b and 5c, unconstrained state-dependent commitment is infeasible (see figure 3c), and hence, all we have to prove is that constrained full commitment is more desirable than full flexibility. The existence condition of constrained full commitment is

$$z^{FF} < \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)} \leq \bar{z}.$$

This immediately means that constrained full commitment is welfare-superior to full flexibility:

$$\underbrace{U\left(z^{FF}, \frac{\beta\delta W^g}{(1-\phi^m)(1+\beta\delta)}\right)}_{\text{full flexibility}} < \underbrace{U\left(\bar{z}, \bar{z}\right)}_{\substack{\text{constrained} \\ \text{full commitment}}} . \tag{A.6}$$

The magnitude relationship from (A.2) to (A.6) proves lemma 5.

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