Edgeworth's formalization of parametric external economies　as a germ of a game theoretic view : What was the hard core of the British Marginal Revolution?

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Abstract

While Marshall introduced the concept, 'external economy', it is said that Chipman(1970) formalized 'parametric economies of scale' in order to reconcile the maximization behavior of individual firms with increasing return to scale of the economy. Chipman's idea promoted the next stage of the development, the endogenous growth theory through Romer (1986) and in this, a new type of capital, knowledge or human capital, has been focused in the driving forces of economic growth.   But who is the true originator of this development and how can we appraise the development in the history of economics?

Though Chipman indicated that his parametric idea is originated from Edgeworth, and H. Cunynghame, their contributions are scarcely known and appraised in the history of economic theory. This paper tries to show that the parametric formulation is intrinsically of Edgeworth and that his aim is not only showing how the individual's maximization behavior can be consistent with the external economy under the competitive system but also trying to suggest the more general framework where individuals are partly conscious of other's consumption and production behavior as seen in the modern game theoretic models.   In this formulation, Edgeworth tried to base the micro theory on a kind of strategic interactive system and to suggest that the structure of the system should not be arbitrarily determined by analyst's preference but should be positively specified by using statistical data.

There is an important message in this on the methodological and theoretical hard core of the British Marginal Revolution. Marshall's external economy and Edgeworth' parametric idea have been, in their understanding, presumed to be synthesized in a broader approach toward market interactions, differing from the general equilibrium type of approach. They have assumed the whole economic structure where the productivity of the economy, therefore economic gains, would be marginalized by the frontier of enveloping structure of each specific technology or preference.   Within the structure, individual economic actions are cooperatively or conflictingly strategic interactive.

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1. Introduction

While A. Marshall constructed the concept of 'external economy', J. Chipman considered 'parametric economies of scale' in Chipman (1965), where he referred H. Cunynghame and F.Y. Edgeworth as the source of the idea. Chipman analyzed how the competitive equilibrium can be derived from a kind of myopic optimization behaviors of individual firms under the external economy. Chipman's formalization has been extended in the endogenous growth theory by P.Romer since 1980's. In this, human capitals such as knowledge and education are focused in the literatures, in which various endogenous economic growth paths beyond the physical constraints are analyzed in the development economics and international trade theory with some implications for the economic analysis and policy. This paper investigates what kind of role Cunynghame and Edgeworth had played in the development of the basic ideas of 'parametric economies of scale', in particular, in relation to Marshall.

The contributions of Edgeworth, and Cunynghame in these literatures are scarcely known and appraised in the history of economic theory. This paper tries to show that the parametric formulation is intrinsically of Edgeworth and it is a very special case within his scope. His aim is not only showing the consistency of the individual behavior under the scale economy with the competitive system but also trying to suggest the more general case where individuals are partly conscious of other's consumption and production behavior.

On the one hand, Cunynghame provided concrete descriptions and geometrical explanations of externalities, in particular, snob effects or conspicuous consumptions, earlier than Veblen's *the theory of Leisure Class.* Cunynghame (1904) gave a direct influence on Leibenstein's bandwagon effect analysis. Edgeworth, on the other hand, interpreted that each individual's myopia that is assumed in the parametric economy is a property of the special and limit case applicable to consumption side of externality. This case is later, in 1980's, extended to the idea of human capital of the endogenous economic growth, the property of which provides the unique interpretive potential that market

behaviors are sometimes negligibly small to count their own effects as seen in social tastes or prevalent knowledge and education of the social standard. However, Edgeworth saw that the myopic behavior on the production side scarcely finds its concrete example and that each producer being conscious of other's economic action, their relations can be analyzed generally in an interactive system rather than as a special case of 'parametric economies of scale' The parametric formulation is Edgeworth's device as a connector between his general scope of interactive markets and Marshall's partial equilibrium analysis under the competitive regime.

The above massage provides a fundamental question, "What was the methodological and theoretical hard core of the British Marginal Revolution, at least for Edgeworth and Marshall?" Marshall's external economy and Edgeworth' parametric idea have been, in their understanding, presumed to be synthesized in a broader approach toward market interactions, differing from the general equilibrium type of approach. They have assumed the whole economic structure where the productivity of the economy, therefore economic gains, would be marginalized by the frontier of enveloping structure of each specific technology or preference. While Edgeworth saw that individual economic actions are cooperatively or conflictingly strategic interactive and the outcome of their action would be cycle, jump or indeterminate, Marshall made the partial equilibrium analysis along his time structure, where his interpretative view for the organic and evolutionary growth of industrial organizations was developed. Edgeworth's parametric formulation connects these two analytical scopes and what is to be remarked is the fact that the former vision precedes the latter in the British Marginal Revolution period.

2. The historical appraisal of Chipman's parametric formulation

"A parametric economy of scale "(Chipman (1970) p. 349)" is a term used by Chipman (1970).

"Entrepreneurs are assumed to believe that his firm is operating under constant return to scale, and any departures from this assumed output-factor relationship are interpreted by him as brought about by a perturbation in his unit-homogeneous production function, even if such departures are caused in part by changes in his own level of output."(Chipman (1970) p. 349)

Chipman explains this interpretation with the development of division of labor at Adam Smith's pin factory. One company uses specialized labor as a result of expanding production and progressing division of labor. Then, such specialized labor will be available

to other companies in the industry. However, since the specialized labor is pooled only when the whole industry is expanded, the contribution of one enterprise to this process is negligibly small. Therefore, it is external to a company and is not recognized. (Chipman (1970) p. 349) In this way, Chipman (1970) has introduced the concept of parametric economies of scale, Chipman (1965) investigating the history of this idea considerably in detail. However, there are few researches in the history of economics to consider the historical background of Chipman (1965)[1].

Boblulescu (2007) and Gehrke (2015) have studies the significance of Chipman's formulation in relation to the modern analysis of increasing return to scale[2]. Bobulescu (2007) evaluates Chipman's "parametric economy of scale" as a logical development under Marshall's tradition. In other words, Chipman's formulation is a logically natural development based on Marshall's original external economy including the cost controversy and the subsequent criticism by Sraffa etc. in the 1920s. However, Bobulescu (2007) is trying to differentiate some of the development in 1920's, such as, J. Viner's pecuniary externality, the monopolistic competition of J. Robinson and E. Chambelin from Chipman's formulation.

Gehrke (2015) took the same theme as Boblulescu (2002, 2007) and gave another evaluation, comparing the formulation of Viner, Chipman and Krugman (Gehrke (2015) P.23.note 2). Gehrke (2015) opposed to positioning the modern formulation on the extension of the logical development of Marshall's tradition (Gehrke (2015) P.25.note 31). Gehrke (2015) emphasizes that on the one hand, the formulation of Chipman is just a matter of technological externality in the general equilibrium while on the other Viner deals with pecuniary type of externality in the partial equilibrium. It appraised that Viner's intention is to convert the context of partial equilibrium into the framework of monopolistic competition theory. (Gehrke (2015) P.3) And it is said that Krugman's formulation is on the extension of Viner's trial, the development of which is interpreted as a positive contribution. On the other hand, the formulation of Chipman is not evaluated as a promising revision of Marshall's framework.

It is written in footnote 27 of Gehrke (2015) as follows.

"Chippman's concept was adopted, among others, by Panagariya (1980), Inoue (1981), Suzuki (1996) and Silvestre (1999), but it cannot be said to have triggered a

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[1] Suzuki (2009) quoted Chipman (1965) and mentioned that Edgeworth is one of souces of the parametric idea.

[2] Bobulescu (2007) does not refer to Chipman (1965) and therefore does not mention Edgeworth and Cunyngham. Gehrke (2015) refers to Edgeworth and Cunyngham with reference to Chipman (1965), but does not consider their content.

particularly broad stream of literature. It was also mentioned, and immediately dismissed, in the opening paragraphs-that is, in the so-called "literature survey" – of Romer's seminal contribution to the new growth theory: Subsequent work demonstrated that it is possible to construct a consistent, general equilibrium models with perfect competition, increasing returns and externalities (see e.g., Chipman (1970)) ... Following Smith, Marshall and Young, most authors justified the existence of increasing returns on the basis of increasing specialization and division of labor. It is now clear that these changes in organization of production cannot be rigorously treated as technical externality. Formally, increased specialization opens new markets and introduces new goods. All producers in the industry may benefit from the introduction of these goods, but they are goods, not technological externalities. (Romer (1986) p. 1005) " (Gehrke (2015) p.25.)

On the one hand, Gehrke interprets that Chipman's concept covers only technological externality with little economic implication to its application. On the other hand, Gehrke (2015) positively focuses on the later development of increasing return to scale phenomena such as spillover effects of knowledge, linkages of industries, in the international economics and the spatial economics.

Together with these evaluations, there are two types of developments in the approach of increasing returns to the modern era. One is (1) Chipman type = technological externalities under the perfect competition, the other is (2) Viner-Krugman type, = pecuniary externalities under the imperfect competition. (1) Chipman type is compatible with the competitive equilibrium. (2) Viner-Krugman type considers some processes in which various consumers and companies and capital goods interactively spread through cumulative propagation under incomplete competition. The problem that this paper considers is how and in what context the original concept of "parametric economy of scale" started in the history of economics. In addition to it, why does Romer type come out from Chipman type? Romer type considers the externality of knowledge / human capital where individual contribution in the economy is negligibly small.

3. H. Cunynghame and his contribution toward the modern externality

According to Cunynghame's biography (Ward and Spencer (1938)), he was equipped with a little academic education, but he was a lawyer who worked vigorously on realistic issues and did the job of a member of the government. He worked for the enactment of the regulation law for workers working in coal mines, traveled to the coal mine himself, gathered various opinions and discussions, and led them. He was a key person who tackles

the reality of Britain after the Industrial Revolution from all angles. Among scientists or artists in London at the turn of the century, probably there is no one who has never met him. For example, Painters Holman Hunt, Whistler, Herbert Spencer, Huxley, William Morris, Wilde and others. He was a "Brain of the town", which connects in various way people and stories that he encountered in society and parties. Also, he surveyed colonial finance going abroad, gave an advice on construction of London's arts school, and mediated Alaska whale ownership problems. It can be seen in the photograph of Cunynghame at the beginning of the biographies, that he is an active person who has a very big head on a squatting petite body. However, in this biography, economic contributions are hardly told. Marshall is mentioned as one of the teachers of the university era and an economic writing 1904 is slightly mentioned. Perhaps there was a lot of involvement with economics, but the share of that would have been negligible in the whole.

In economics related literature, mentioning Cunynghame's person is the comment by Whitaker who wrote Cunynghame's section of Palgrave Dictionary and Keynes' Cunynghame memorial service (Keynes (1935)). The following *Times* died article quoted by Keynes is interesting.

"It may be doubted whether any important official position has ever been held by an odder or, in some ways, a more remarkable personality than Cunynghame. Of his cleverness there never could be any doubt, though the routine of a public department was hardly the best field for its display. There appeared to be no subject of which he had not at least a working knowledge, and certainly none of which he was not prepared to talk at large. He was a practical electrician, and at one time had been vice-president of the Institution of Electrical Engineers; in his house were installed a laboratory and workshop; and official interviews would be enlivened with disquisitions upon Hegelian philosophy and analogies drawn from the novels of Balzac (the plots of which he seemed to know by heart), the medieval Schoolmen, or indeed any other branch of literature or learning upon which his nimble intelligence had recently been engaged. The story goes that when " phossy-jaw," contracted in the manufacture of matches, was engaging the attention of the Home Office, Cunynghame announced that he had himself discovered and manufactured a non-phosphoric safety match. A meeting in the Home Secretary's room was accordingly arranged for the testing of this invention, but the future prospects of the Cunynghame match went up in the smoke of the explosion that ensued. On another occasion, when the prevention of miners' phthisis was the subject of inquiry by the Royal Commission on Mines, he astonished his colleagues by the production of a

model man on a large scale which he had made in his own workshop, showing the working of the throat and lungs and how dust was absorbed into the system. These incidents are typical of this unconventional public servant, who, with his wide learning and practical knowledge of many departments of life, might, if he had concentrated more, have been one of the great departmental heads of his generation. As it was, his official career was less successful than that of many of his intellectual inferiors, and to his disappointment he was not made head of the Home Office when Sir Mackenzie Chalmers retired in 1908." (Keynes (1935) p.401, *The Times* (1935))

The explosion incident in the article is as if it is a caricature scene, reminiscent of an eccentric inventor. Cunynghame is also inventing tools in economics. He entered St. John's College in Cambridge University in 1870 and was under the influence of Marshall, who was appointed as a lecturer of the moral science. According to Mrs. Marshall's letter to Keynes, Cunynghame was one of Marshall's favorite disciples in the early 70's. And while attending Marshall's lecture, Cunynghame invented a creative instrument to draw a right-angle hyperbola, seeing that Marshall was in trouble drawing the curve on the blackboard. The tool was lost now, but it was exhibited at the Cambridge Science Society. Marshall published a paper to explain its usage and analyzed its monopolistic value in the paper[3].

Among Cunynghame's socializing with cultural people and key persons, there were also mathematical economists in the marginal revolution, such as Edgeworth or P.H. Wicksteed. It is known that Jevons and Edgeworth lived in Hampstead, a suburban residential area in London, as a neighbor, and had interactions such as hiking, cycling, and study sessions. Cunynghame's unpublished thesis of the 1880's was delivered at the study meeting of Hampstead. The meeting was called the Economic Circle of Wicksteed and was held at Hamstead's Beaton house (Ramée Beeton) once every two weeks, and Cunynghame, Edgeworth, Shaw (G.B. Shaw), Foxwell (H.S. Foxwell), Web (S. Webb) participated, and Marshall also occasionally participated[4].

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[3] "The machine, which Mrs. Marshall mentions, for drawing rectangular hyperbolas on the black-board, was famous thereafter to forty generations of undergraduates attending Marshall's lectures. It was communicated to the Cambridge Philosophical Society along with Marshall's first original contribution to economics (and his second appearance in print)-'Graphic representation by aid of a series of Hyperbolas of some Economic Problems having reference to Monopolies,' *Proceedings of the Cambridge Philosophical Society*, Oct. 1873." Keynes (1935) p.400.
[4] Refer to Whitaker (1996) vol. 1, p.255 Letter no.224 from Marshall to Foxwell. Whitaker (1987) p.739. Howey (1960) pp.118-30. Barbé, L. (2010) p.116.

The economic contribution of Cunynghame can be seen from Keynes' biographical memoirs and Edgeworth's book review.   One of them is to illustrate and generalize the demand curve by 'successive utility curves', the generalized supply curve, what is called 'successive cost curves'. Discussions on this successive utility curve, successive cost curve have been developed in Cunynghame (1892), but they have not been developed in Cunynghame (1904). Cunynghame had withdrawn them from his main contribution in Cunynghame (1904)

First, Cunynghame mentioned a case where the externalities of demand, in particular the consumption of other consumers, influences the utility obtained from his consumption.

"It affords a man who is eating bread no satisfaction to know that his neighbour has got none; but almost the whole value of strawberries in March, to those who like this tasteless mode of ostentation, is the fact that others cannot get them. As my landlady once remarked, ' Surely, sir, you would not like anything so common and cheap as a fresh herring?'   The demand for diamonds, rubies, and sapphires is another example of this. As the number increases, not only does the price go down, but the very pleasure of those who already have them is decreased by their becoming common." (Cunynghame (1892), p. 37)

This discussion can be evaluated as a pioneering one of Veblen's conspicuous consumption or snob effect as seen in "The Theory of the Leisure Class" in 1899. More directly, it is positioned as a prior study of the bandwagon effect in Leibenstein (1950). Leinenstein refers to Cunynghame (1892) at P.185 of Leibenstein (1950).   Both are common in that they try to show by means of a diagram that the aggregate demand function composed of individual demand curves has a upward sloping part.

It is worthwhile to note what motivated Leibenstein (1950), "Bandwagon, Snob, and Veblen Effects in the theory of Consumers' demand".   His enquiry was suggested by O. Morgenstern as the "non-additivity" problem, where market demand curve is not the lateral summation of the individual demand curve.   This phenomenon is seen, for example, in the collective demand curve with bandwagon, snobs and Veblen effects.   And Leibenstein slightly mentions that the phenomenon also applies to the collective supply curves.   While Morgenstern imply that "since coalitions are bound to be important in this area only the "Theory of Games" (developed by Von Neumann and Morgenstern) is likely to give an adequate solution to this problem," (Leibenstein (1950) p.183), Leibensteain tried to analyze through the use of conventional method by graphical demand curves, because he thought he is not competent to judge whether the game theory is proper to the analytical

tool. It is notable that the "non-additivity" problem is originally related to the market view based on strategic interactions of the game theory.

Furthermore, Leibenstein refers to Cunynghame and his 1892 paper as one of the past literatures in this line of research. He explains the reason why the interpersonal effect in the market has been ignored in the current texts as follows.

"One reason why the interpersonal effects on demand have been ignored in current texts may be the fact that Marshall did not consider the matter in his *Principles*. We now, however, from Marshall's correspondence (note: Pigou, *Memorials of Alfred Marshall*, pp.433 and 450. These are Marshall's letters to Pigou and Cunynghame which indicate that Marshall had read the articles (*EJ*. 1892 and *E.J.* 1903), where Pigou and Cunynghame consider the matter.), that he was aware of the problem." (Leibenstein (1950) p.186.)

The above indicates that Leibenstein's theme is placed on the extension of Cunynghame (1892). At the same time, Leibenstein remarked that though Marshall knew the content of the paper, he did not consider the interpersonal effects on demand in his *Principles*. Leinbenstein's bandwagon analysis could be placed on the half way to the strategic interactive market view in the history of the concept, one of origins of which is Cunynghame (1892). Marshall may have intentionally avoided the way.

The following is a diagram of the demand curve that is derived from a group of successive utility curves, one of which is designated, for example, as $\Delta\delta$ and is negative to the axis OX. Cunynghame explains that the demand curve will be in general negative, but that in rare instances it might at the commencement be positive as in the Figure 4 (Cunynghame (1892) p.40.)

Fig. 4.
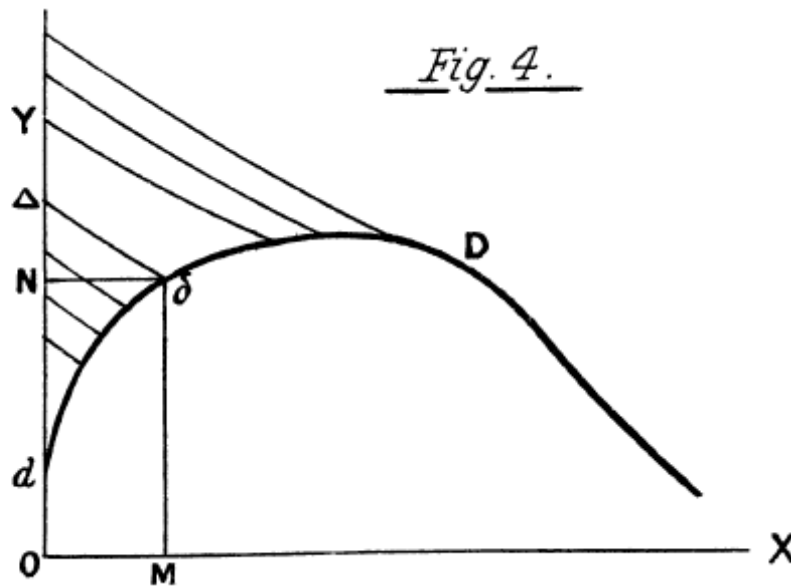
This case is, according to Cunynghame, as when a particular edition of hymns became useful in direct proportion as it was widely used. It is the case that there is positive externality among users in the society as Leibenstein's bandwagon effects or Romer's knowledge capital. Edgeworth explains the relationship between the individual demand curve and the aggregate demand curve in a form to be seen later. Though Cunynghame's description has ambiguity and not necessarily given a consistent explanation as to the relation of groups of successive utility curves with the demand curve.

However, Cunynghame is clear in asserting that a group of successive curves does not represent states of some lapse of time but a state at one time.

"But a group of successive curves is the expression of a state of facts existing at one time, viz. a set of hypothetical consumers' values, and not a group of successive time phenomena." (Cunynghame (1892) p.39.)

Individual demand curves depend on some social conditions, for example, the state where "strawberry is scarce", the state where "bread is common", etc. Though the social condition can be resolved into individual demand curves as Leibenstein analyzed, Cunynghame does not provide consistent explanations.[5]

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[5] Leibenstein tried to construct aggregate demand curves with external consumption effect, such as bandwagon effects, snob effects and Veblen effects. He indicates that under the diminishing marginal external consumption effect, there is a stable collective demand curves.

Cunynghame keeps track of the decline of cost reduction of the supply side in the similar structure to the demand side.

"Just as the demand value varies with the amount purchased, so also will the supply-cost, or, in other words, the cost of the manufacture of an article estimated in money vary with the amount supplied, and sometimes this cost will rise, sometimes it will fall. The phenomenon of an increasing cost of production is illustrated generally by agricultural produce, and raw articles, of which the difficulty of production increases with the amount produced. On the other hand, the cost of production of manufactured articles generally diminishes with the amount produced on account of the greater facilities for the use of machinery and other economy of production which is afforded by a large output. "(Cunynghame (1892), p. 39-40)
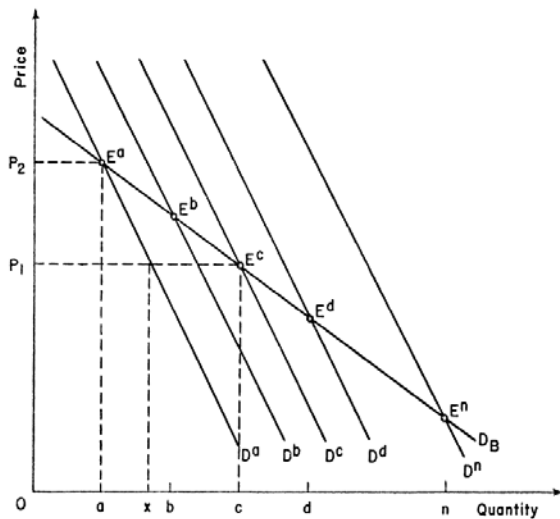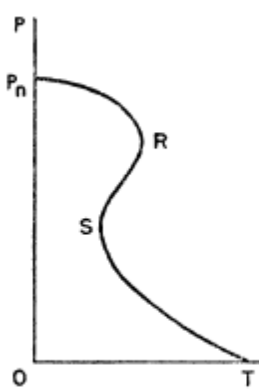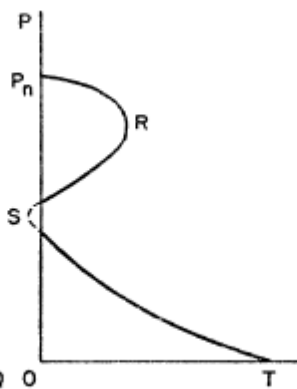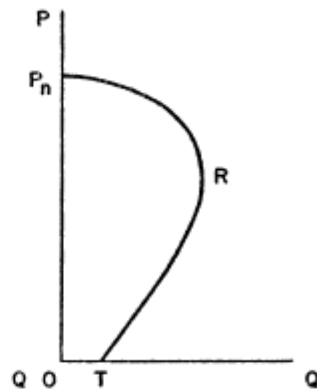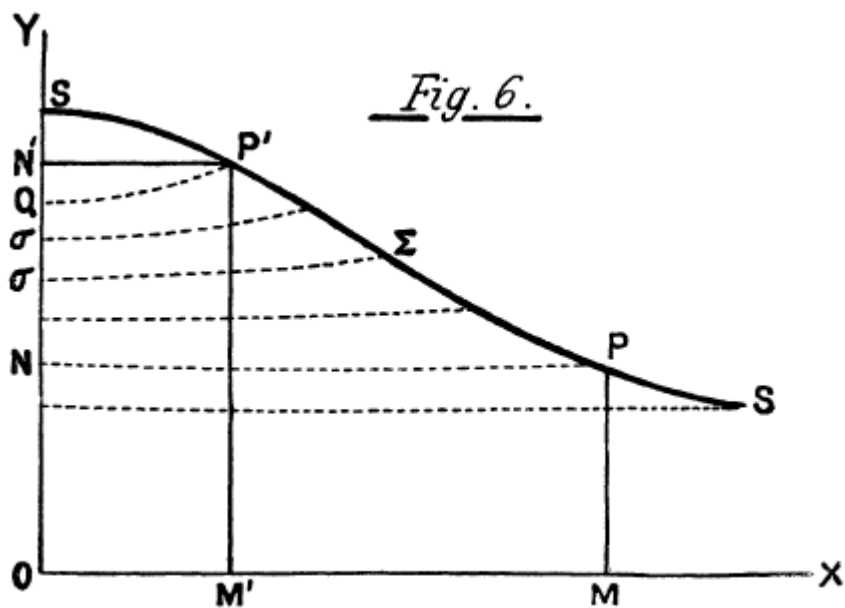

FIGURE 1

FIGURE 5A

FIGURE 5B

FIGURE 5C

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Fig. 6.

The production cost curve (dotted line), which depends on the conditions of economy available for production, such as "the workers of related work are exhausted" and "there is a limit to the necessary means of transportation".[6]   A series of points on the cost curves is a supply curve that slopes downward to the right.

Overall, Cunynghame's achievement can be evaluated as intuitively showing how to configure sequentially the demand curve that depends on state as the state changes.   While his graphical explanation is too naïve to hold the consistent analysis, the framework of the state dependent demand curve or supply curve is an important step toward the insight of the strategic interactive view for the market.   Based on this intuitive insight, it can be said that Leibenstein considered the problem of constructing the aggregate demand curve from the individual demand curve when there is consumer's externality under some necessary conditions.

4.   Cunynghame, Marshall and Edgeworth; what is their opposition?

In the background that Cunynghame (1905) book review was written, what kind of discussion had Marshall, Edgeworth and Cunynghame developed on their analysis of

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[6] Cunynghame states as follows in Cunynghame (1904) p.90."Various causes prevent the full effect of the instability of manufacturing prices being felt.   The necessity of carriage from one place to another is an obstacle.   The impossibility of suddenly creating the necessary skilled labour is another; timidity in enterprise, and the persistence of old habits is another; and of course protective tariffs would also help.   The intervention of a sort of arbitration board, to retard sudden transformations, would possibly have some effect, if it were not probable that the Problem is so complicated that the medicine might produce worse effects that the disease."

'successive utility curves' and 'successive cost curves'?   Why had Cunynghame withdrawn them from his main contribution in Cunynghame (1904).

It is known that there were serious conflicts between Edgeworth and Marshall and that Cunyghame (1892) was a trigger to the conflict.   In a letter to Edgeworth, Marshall reckons Edgeworth's discussion as "Murder" while discussing Cunningham's analysis. Concerning this letter, contemporaries like Keynes pointed out, suggesting that there was a serious confrontation between them. (Keynes (1935) p.404)

Marshall wrote to Edgeworth on 26 April 1892.

"I cannot refrain when I see you plunging it into a medium in wh it cannot breathe, from calling MURDER!" (Whitaker (1996) p.69, Letter 406)

Marshall took up the discussion of Cunynghame (1892), develops an objection, and finally writes to shout this sentence. In this letter, Marshall is asking the following question.

"Let y=f(x) be the equation to one of Cunynghame's successive cost curves: What does the y mean, & what does the x mean?"

Cunynghame may have withdrawn his successive cost curves from Cunynghame (1904), probably confronted with Marshall's sever criticism.   However, despite being a book review to Cunynghame (1904), Edgeworth has reconstructed this issue taking up the successive utility curve and the successive cost curve of Cunynghame (1892).   It is inferred that Edgeworth's aim of the book review is to discuss these parts and answer Marshall's question, "What does the y mean, & what does the x mean?"

Apart from Edgeworth's interpretation, how Marshall himself treated this problem in *the Principles* as the edition goes from the first to the eighth edition.   Guillebaud's comparison of the editions elucidated in the eighth edition does not necessarily let us know how Marshall had changed.   The following is the comparison through editions only concerning with Marshall's mention to Cunynghame.

There are three mentions to Cunynghame in *the Principles.* (1) Cunynghame's instrument for drawing hyperbolas (2) Cunynghame's private circulated paper   (3) Cunynghame (1892) in *Economic Journal*.   The years in which each edition was published are the following.   The first edition in 1890, the second edition in 1891, third edition I 1895, the fourth edition in 1898, fifth edition in 1907, sixth edition in 1910, seventh edition in 1916 and eighth edition in 1920.

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As to (1), in the preface for the first edition, there was an acknowledgement expressing Marshall's thanks to Cunynghame for his making the instrument for drawing hyperbolas. This mention reflects their close and friendly relationship. This preface for the first edition has been inserted from the second to fourth edition in the same form (p. xv n.1 for the second and third edition and p. xiii for the 4th edition). However, though the preface for the first edition is printed in the following edition (from fifth edition to eighth edition), the mention to Cunynghame was omitted.

As to (2), from the first edition, the following statement has been inserted.

"If the change is gradual, the supply curve will assume in succession a series of positions, each of which is a little below the preceding one; and in this way we might have represented the effects of that gradual improvement of industrial organization which arises from an increase in the scale of production, and which we have represented by assigning to it an influence upon the supply price for long-period curves. In an ingenious paper privately printed by Mr H. Cunynghame, a suggestion is made, which seems to come in effect to proposing that a long-period supply curve should be regarded as in some manner representing a series of short-period curves; each of these curves would assume throughout its whole length that development of industrial organization which properly belongs to the scale of production represented by the distance from Oy of the point in which that curve cuts the long-period supply curve (compare Appendix H, 3) and similarly with regard to demand." (Marshall (1997) p.383-384)

The above is Marshall's interpretation of Cunynghame's successive cost curve which expresses the effects of increasing return to scale as the development of industrial organizations. Marshall clearly states that the effects are graphically showed as the relation between a series of short-period supply curves and the log period curves. This interpretation is based on Marshall's consistent modeling by partial equilibrium analysis with time period differentiation, mainly short-run and long-run.

As to (3), from the fourth edition there appears the following statement in the Appendix H.
"One difficulty arises from the fact that a suitable time to allow for the introduction of the economies appertaining to one increase in the scale of production is not long enough for another and larger increase, so we must fix on some fairly

long time ahead, which is likely to be indicated by the special problem in hand, and adjust the whole series of supply prices to it.

We could get much nearer to nature if we allowed ourselves a more complex illustration. We might take a series of curves, of which the first allowed for the economies one year, a second curve doing the same for two years, a third for three years, and so on. Cutting them out of cardboard and standing them up side by side, we should obtain a *surface*, of which the three dimensions represented amount, price, and time respectively. If we had marked on each curve the point corresponding to that amount which, so far as can be foreseen, seems likely to be the normal amount for the year to that curve would be a fairly true long-period normal supply curve for a commodity obeying the law of increasing return. Compare an article by Mr. Cunynghame, in the *Economic Journal* for 1892." (Marshall (1997) P.667 n.)

Marshall subdivides the time structure by introducing economies of one year, the same for two years, for three years etc. in this statement. What is remarkable is that in spite of the clear assertion by Cunynghame indicated above, "a group of successive curves is the expression of a state of facts existing at one time", Marshall never concede that the scale economy can be analyzed in the model of a state of facts existing at one time. He presupposes that each individual firm or consumer behave within the time structure, while effects of increasing return to scale can be introduced within some time period enough appertaining to it. The interactions of behaviors firms and consumers at one time were not considered. Why? What is the reason of Marshall's excitement when he used the word, 'Murder', in his letter to Edgeworth?

It should be clarified that Marshall and Edgeworth were conflicting in their analysis of market equilibrium under increasing returns with an opposition between the two methods of economic analysis. Nakano (2012) (2013) (2015) elucidated the opposition between Marshall and Edgeworth, whereas Edgeworth has been interpreted under the Marshall's umbrella except for the technical arguments on the barter process, called the barter controversy. Their opposition will be explained in the following as long as it makes it easier to understand the background of their discussion of this paper.

The following figure is inserted on a page of *Nature,* Edgeworth (1890), which is his book review for the first edition of Marshall's *Principle.* Edgeworth indicates Marshall's misrepresentation of the situation where there are multiple equilibria under the increasing return to scale on the supply side. Edgeworth states that an individual firm acting on the declining supply curve would not maximize his profit.
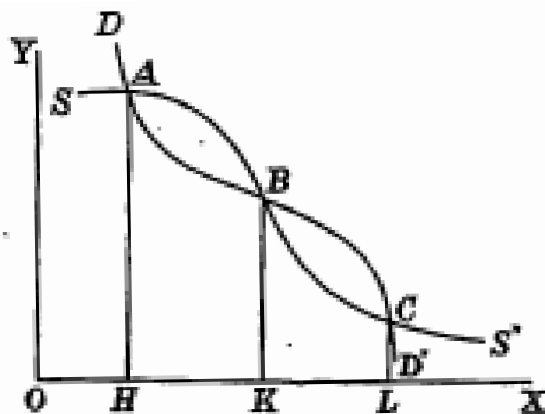
Edgeworth (1890) *Nature* p.363

When Edgeworth criticizes the above Marshall's partial equilibrium analysis in his *Principle*, he has the offer curve analysis in Marshall (1879) in his mind. Particularly diagrams (figures 5, 9) concerning the movement of trade equilibrium when shifting from low production efficiency technology to high technology in domestic industry. This is an analysis of how the economic equilibrium using high technology appears in the economic environment potentially having increasing production technology[7].

In other words, the analysis concerns how a trade equilibrium shifts from A through B to C in figure 5 when increasing return on the scale is brought about through expansion of the domestic market or expansion of foreign markets. Marshall refers to the concept of Smith's "the extent of the market" and explains how the phenomenon that the market size reduces the relative price of imported goods and export goods while expanding trade transactions. Therefore, we assume crossover of offer curves as shown in figure 5 by the setting "Decrease in the value of export goods, the amount of imported goods to be exchanged for export goods will decrease". When OE is the home offering curve and the partner country is OG, the horizontal axis shows the export amount x and the vertical axis shows the import amount y from the partner country.[8]

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[7] Marshall explained that the contents of figures 5 and 9 are the "Class II" problem of the exceptional case of trade problems. "give the name of 'Class I.' to the second exceptional case to which reference has been made; the case namely, in which an increase in the amount of wares which a country produces for exportation effects a very great diminution in the expenses at which she can produce them; so that the consequent fall in their value diminishes the total amount of the imports that she receives in exchange for them. (Marshall (1879) p.5-6.) "The case has its origin in the fact that the wares which a country exports maybe such that the difficulty of producing them diminishes very rapidly when their amount increases. It is indeed true, as has been said, that in general the production of a commodity on a large scale for home consumption precedes the development of any considerable foreign trade in it. Still the extent to which division of labour in the production of it can be carried, is onlarged by every extension of the foreign markets for it." (Marshall (1879) p.12-13.)

[8] Marshall does not write P and Q in the figure, but according to Marshall's explanation, the point where a straight line going vertically from M crosses OE is called P and the point vertically rising from R is confronted with OE can be regarded as Q . Marshall says as follows. "Continue to interpret the problem of Class II in the figure, P and Q (figure 5) are two points on O and E, PM and QR are perpendicular to Ox, QR is larger than

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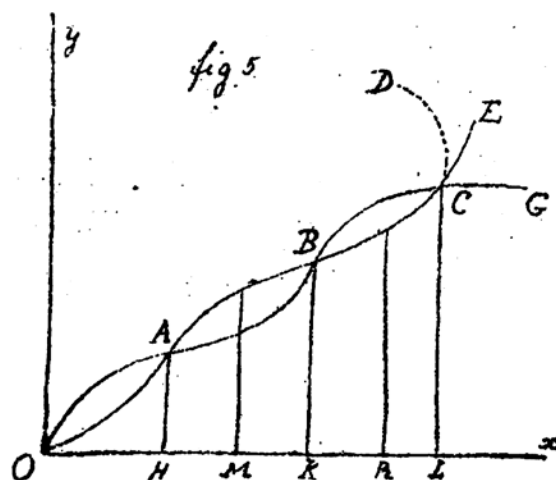
Marshall (1879) figure 5

Edgeworth highly appraises the analysis of Marshall (1879) figure 5, because it presupposes envelop structures of various technologies with increasing return to scale and depicts a dynamic competitive process through which traders or firms compete each other to dominate the market by watching their competitor's actions. There is a possibility that Edgeworth could have been influenced by this analysis when he developed his game theoretic view in his *Mathematical Psychics,* which will be discussed in another paper in detail. However, Marshall omitted this analysis from his *Principles* and replaced it with the partial equilibrium analysis by the figure above (Edgeworth (1890) *Nature* p.363). Accordingly, Edgeworth trial in *Mathematical Psychics* based on the analysis of Marshall (1879) has been lost importance in *Principles.* This is the background of their opposition that lies in the parametric economies idea.

5. Edgeworth and his formalization of 'parametric economies of scale'

Edgeworth puts a book review on Cunynghame's 1904 book in the Economic Journal in 1905. However, rather than mentioning only 1904 books, the mention of 1892 papers is

PM In case and Class I case, the ratio of QR to OR must be greater than the ratio of PM to OM (Proposition II) However in Class II, the increase in the amount of fabric produced in the UK is 1 As the production cost per yard is greatly reduced, the increase in imported linen will reduce the value of 1 yard linen in the UK, as well as allowing more linen to be purchased at each yard of linen than before. Therefore, the increase in the export of dough will increase by a large proportion rather than the smaller proportion than the import of linen. "(Marshall (1879) p.14.) In other words, because economies of scale work, the situation that the price of export goods for imported goods become cheap when trade expands is caught in the shape of the offer curve

the main object of his review. On the one hand, it is pointed out that using real economic case and the metaphor, Cunynghame's explanation is attractive to the scholars of beginners. On the other hand, Edgeworth confirms that it is insufficient by merely explaining with the figure, citing Cunynghame's statement, "The figure cannot determine the issue of free trade versus tariffs, the figure only shows the status of the problem" (Cunynghame, (1904) p. 102), at the beginning of his review.   Furthermore, indicating it is difficult to combine properly both concrete explanations and scientific explorations, Edgeworth evaluates that the balance between them is not well enough in Cunynghame's discussion.

   "We had hoped that the path struck out by him in his original article on "Exchange Value" in an early number of the Economic Journal (1892) would have been now converted by him into a high-road accessible to the wayfarers of science, even though not specialists. But this hope has not been fully gratified.   There still, as it appears to us, remains some difficulty, which we shall endeavor to smooth over by a free restatement." (Edgeworth (1905) pp.136-137)

As is stated above, the free restatement is one of the objects of Edgeworth's book review. In other words, although Edgeworth was hoping Cunynghame to reformulate the paper in 1892, as it was not yet done in the 1904 book, Edgeworth embarked on it in the review paper.

Edgeworth draws the following quote from Cunynghame's book at the beginning of the book review.

   "Two manufacturing rivals are like men pulling against one another on a rope, one on each side of the summit of a hill. When one of them is once pulled over the summit the other can run away with him. But if the men were pulling on each side of a hollow, as in rival production of the agricultural order, then when one got pulled down a little his opponent's task would become harder; so they would come at last like a marble in a bowl to a position of equilibrium." (Edgeworth (1905) p.136. Cunynghame (1904) p.89)

In the book of Cunynghame, the above quotation is followed by the statement, "The points on the downward supply curve is unstable, theoretically, it will be produced by the cheapest producers and countries ".   In other words, monopoly will occur if there is an increasing return to scale, but if there is some friction in reality, there is a possibility of antagonizing the competitors.   For example, some cases are enumerated, for example,

where industrial leaders retire, where competitors are exhausted to make money, where the workers of related work are exhausted, and where there is a limit to the necessary transportation means. Some of these are concerned with the internal economy of each enterprise and some can be interpreted as external economies. However, Cunynghame did not develop the theory to explain how these frictional factors derive the downward sloping supply curve.

Why did Edgeworth quote a metaphor comparing to the situation where the above rival enterprises have tug-of-war across the summit of the mountain at the beginning of his review? It is inferred that the view that captures unstable competition under increasing returns is the key for their opposition among Marshall, Cunynghame and Edgeworth. According to Guillebaud, a relative of Marshall, who edited Marshall's *Principles*, states in a retrospective review of Marshall that Marshall often says about Edgeworth as follows.

"Apropos of Edgeworth, for whom he had a strong personal liking and whose ability as an economics he rated highly, he said, 'I would illustrate his way of dealing with an economic problem as follows: "let us assume" says Edgeworth "that we have two elephants suspended from the end of a rope. Give the elephants a push and then, disregarding the weight of the elephants, work out what happens to the rope.' "
(Guillebaud (1971) p.7)

In other words, Marshall cynically told Guillebaud that the characteristics of Edgeworth's economic analysis lie in instability of the firm's competition under increasing returns. It can be interpreted that Marshall, at least after publishing *Principle* in1890, had not base his analysis on the micro-instability of the competitive process between firms. Marshall thought that the time period for economic activities are basically divided into the short-run and the long-run, and that the economic process in the long run can be theoretically traced through the partial equilibrium in the short run. Marshall interprets that the increasing return to scale is due to the external economy uncontrollable for individual firm, which develops through the complication of industrial organizations. Marshall, at least in his *Principles*, opposed to Edgeworth, who thinks that competition between firms under increasing returns is unstable, as shown below by indicating the logic to handle the parametric economy to handle the external economy as a special case, Edgeworth tried to take the case of the micro instability actively.

In Edgeworth (1905), Edgeworth picks up the parametric economy as a case of externalities in consumption at first. After that, Edgeworth explains the logical structure equivalent to the externality of consumption as the externality of production. However,

as an interpretation for the real economy, Edgeworth considers that there are rare cases in which the externality of production can be interpreted through the myopic behaviors of individual firm, rather considers that in the most cases where the oligopolistic market or monopoly progresses, each firm is conscious of their behaviors in the market.

Edgeworth starts with the theory of demand and lists the case of "orchid". When orchids are rare, demand increases with rising price of orchids, but as orchids become common flowers, demand will not increase with price rise, only with price decline demand will increase.   In this way, whether or not the consumption of orchids of other people is small or large affects the consumption behavior of individuals.   Within the normal partial equilibrium analysis, Edgeworth explains Cunynghame's illustration as follows.

The demand function of r-th consumers within n persons is set as follows.

$$\xi_r = A_r - B_r p - C_r X'^2$$

$A_r, B_r, C_r > 0, p$ is the price and X is the general consumption level.

As for r＝1, 2, .., n   The aggregate demand function that adds up to all n individuals is as follows.

$$\sum_{r=1}^{n} \xi_r = \sum_{r=1}^{n} a_r - \sum_{r=1}^{n} b_r \cdot p - \sum_{r=1}^{n} c_r \cdot X'^2$$

When $A = \sum_{r=1}^{n} a_r$, $B = \sum_{r=1}^{n} b_r$, $C = \sum_{r=1}^{n} c_r$,   the aggregate demand function is as follows.

$$x = A - Bp - Cx'^2$$

If X′ is not the constant but the same variable as X,

$$X = A - Bp - CX^2$$

$$Bp = -C(X + \frac{1}{2}/_C)^2 + A + \frac{1}{4}/_C$$

The aggregate demand function is in the form of a parabola as above, and X and P are placed in the positive quadrant according to the way of giving constants of A, B, C.   In the positive quadrant, (1) a case where P goes down along X goes the right and (2) There may be a case where both the rising part and the descending part exist.   Edgeworth thinks that the case (2) is a case where demand rises with price rise and external economy of consumption is occurring as the Figure 4 in Cunynghame (1904).   Individual consumers consider the demand volume of the market as a whole as a given amount, if their influence is negligibly small with respect to the aggregate demand in the competitive market. Therefore, X' is considered as a constant.   This myopia of individual consumer is an idea of parametric economy inherited by Chipman. It is pointed out that Edgeworth is actually

reluctant to apply this idea to the supply side. Edgeworth says that this parametric externality concept can be logically developed in a similar way for the external economy of production, but Edgeworth also points out that there are few cases where production actually applies to production.

"The liability of an industry to be monopolized when it obeys the law of increasing returns creates peculiar difficulty in the application of the geometrical method to supply. In order that the theory which has been above set forth with reference to demand should be extended to supply, it must be postulated that the output of each producer is small in comparison with the collective output of all his competitors. But this postulate is apt not to be adequately fulfilled in modern manufacturing industry; as Mr. Cunynghame reminds us in many a striking passage (pp. 79, 86-69)." (Edgeworth (1905) p.141)

In the corresponding part, a monopoly case is compared to the beggar-my neighbor of playing cards, where a monopolist eradicates the market (Cunynghame (1904) p.79). Cunynghame depicted an unstable process of market competition. (Cunynghame (1904) p.89.)

Edgeworth said that, either in case of externality of consumers or that of producer, the case can be logically treated as a parametric economy as its individual demand or supply is negligibly small compared to the total as a whole in a competitive market. However, Edgeworth does not explicitly lead the competitive equilibrium in the case where such an external economy exists. It can be said that the development of this part is the contribution of Chipman (1970). Edgeworth only refers to the shape of the demand curve, on the supposition that the demand and supply curves in the partial equilibrium intersect.

However, Edgeworth is reluctant to apply the parametric economy on the supply side as seen above, would rather keep in mind the process in which individual suppliers react according to aggregate production volume. Edgeworth insists that demand curves and supply curves dealt with in discussion of parametric externality are short-term curves in terms of short-term and long-term classification of Marshall usage as follows.

"They are "short-period" curves because when there occurs a change in the conditions of supply, and accordingly a new point of intersection between the new supply curve and the old " successive utility curve, then—the correspondence between our x and x' being disturbed—the successive utility curve mast be conceived as changing its form until x and x' once more coincide. No such change of form is

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suffered by the proper, or, as we should like to say, "long-period" demand curve. We should have said so if Mr. Cunynghame had not expressly repudiated this nomenclature." (Edgeworth (1905) p.142)

It can be seen that Edgeworth presupposes the process where individual behavior given parameters respond to changes in the total amount. Taking micro instability and its concrete description by Cunningham into consideration, Edgeworth mostly has the oligopolistic market in mind. For that reason, he insists that the supply curve based on the successive cost curves is of the 'short-period'. Considering the case where the behaviors among firms are strategically interdependent, the interdependence can occur in the short-period not in the long-period. This point is crucial in interpreting what is the opposition between Marshall and Edgeworth. Marshall never takes into the analytical framework the situations where individual firms are strategically interdepend in the short-period or at a time.

The extension of Leibenstein and the development of Romer are basically concerned with externalities of social levels of consumption and available standards of living including knowledge and education. This development is considered to be consistent with Edgeworth's parametric idea applicable for the common goods augmented by negligibly small agent behaviors. But this extension is the special case in which Edgeworth tried to find a point of contact between Marshall's approach and Edgeworth own.

Edgeworth, on the one hand, tries to clarify in what cases the external economy is compatible with the competitive market, but on the other hand, he mainly targeted in Edgeworth (1905) to reveal a new methodological stance how to analyze not necessarily competitive markets. The following two points concerns the methodology. Firstly, as a alternative to Marshallian method, the partial equilibrium analysis with time-period structure, Edgeworth pointed out a method called Collocation, which is J.S. Mill's discussion. Secondly, while referring to G. Barkley's epistemology, Edgeworth pointed out the importance of constructing abstract mathematical models with checking on positive data.

First of all, Edgeworth insists that Cunynghame type of supply and demand curves, that is, the state dependent supply and demand curve, are reasonable as the usual type of supply and demand curves are conceived as price dependent functions. Cunynghame presupposes that there are two periods when orchids are rare and common, and then consumers are exposed to different periods. Edgeworth emphasizes Cunynghame's point that the change in these two periods is not a historical event and that these universal states can be introduced into economic analysis by using the way of thinking, 'collocation'.

"He demands, as we understand, that the new collocation should itself be explicable by conditions which are pre-existent and co-existent in much the same sense as the dispositions represented by the ordinary demand curve. It is thus that we interpret his doctrine: "a group of successive curves is the expression of a state of facts existing at one time, and is not a group of successive time phenomena" (Economic Journal II., p. 39). We understand that there is here predicted the same sort of permanence as that which belongs to the state of facts designated by an individual's demand for a commodity, say tea: the law of demand does not change when the price changes." (Edgeworth (1905) p.137)

Edgeworth claims that a model can be constructed as individual demand behavior and supply behaviors are dependent on aggregate behaviors as well as price. In other words, this discussion describes a methodological basis for modeling where individual player behaves dependent on other player's behaviors.

The original idea of "collocation" in Mill's *A System of Logic* is as follows.

"It is necessary here to remark, that in this resolution of the law of a complex effect, the laws of which it is compounded are not the only elements. It is resolved into the laws of the separate causes, together with the fact of their coexistence. The one is as essential an ingredient as the other whether the object be to discover the law of the effect, or only to explain it. To deduce the laws of the heavenly motions, we require not only to know the law of a rectilinear and that of a gravitative force, but the existence of both these forces in the celestial regions, and even their relative amount. The complex laws of causation are thus resolved into two distinct kinds of elements: the one, simpler laws of causation, the other (in the aptly selected language of Dr. Chalmers) collocations; the collocations consisting in the existence of certain agents or powers, in certain circumstances of place and time." (Mill (1862) p.320-321)

In the above, collocation is explained in the laws of the heavenly motions. As the physical mechanics are resolved into complex combinations of certain agents or powers in certain circumstances of place and time, the complex law of markets, "new collocation", is not only explained by the causal relation with price but also the interdependence of individual demand and supply behaviors in the circumstances of the place and time. This can be understood as a discussion about the methodological basis for making a market

model. In other words, it can be interpreted as an expression for the idea of a model building in a way that economic agents are strategic interdependent".

In addition, Edgeworth expresses that mathematical formulation is more suitable for the complicated modeling than the graphical method, which Cunynghame and Marshall prefer. Because mathematical formulation makes it possible to check in what extent the model fits some various discrete data that are fragmental reflection of the real market. Edgeworth presupposes the statistical analysis, some of which he had contributed. He also interprets that the diagram method can be arbitrary because it depends on freehand writing of analyst.

"Nor are we convinced by the following objection:—"To express an experimental supply curve, as, for instance, Fig. 27, or price of getting coal. Pig. 36, or of producing a book, Pig. 27, or still more the curves of demand for corn or sugar by such an expression as

$$y=f(x)$$

is to invest these curves with an apparently simple law-determined character that they do not really possess." But as all that is knowable—much more than is usually known—is a set of discrete data, so much commodity corresponding to such a price, whether is it more arbitrary to draw a freehand curve through points representing those data, or to use a form which stands for any one of an indefinite number of equations each representing a curve passing through the given points? [In particular, a rational algebraical function $y - A_0 + A_1 x + A_2 x^2 + \cdots + A_m x''$; whether the constants are considered as numerous as the observations and so fitting them exactly, or less numerous, fitting the observations as well as possible.] " (Edgeworth (1905) p.144)

On that basis, Edgeworth insists that formulating a problem with mathematical functional form is of lower arbitrariness and that it can prove more general properties than drawing a concrete curve. He explains using Euclid's geometric proof as an example. Edgeworth says that if you draw a diagram of a specific triangle and prove it using the specific properties of that figure, it is hard to say that it is a general proof. And quoted from Berkeley's philosophical writings as follows. "it is true that the diagram I have in view includes all these particulars, but then there is not the least mention made of them in the proof of the proposition." (Berkeley (1994), p. 248.) Edgeworth concludes as follows. "It seems to us quite tenable that the indefinite symbol "f" obtrudes particularity even less than a concrete curve-line." (Edgeworth (1905) p.144) These methodical discussions can be interpreted as answering Marshall's question in the his letter, "Let y=f(x) be the equation to one of Cunynghame's successive cost curves: What does the y mean, & what does the x

mean?"

In this way, Edgeworth presented the parametric economy mathematically formulated. Its methodological view is trying to open up a way to mathematical analysis and empirical analysis toward a kind of strategic interactive market view. In Edgeworth's argument, we need to understand not only the idea of the consistency of the competitive equilibrium with the externality, but also the different approaches between Marshall and Edgeworth.   The possibility that the game theoretic view precedes the Marshallian partial equilibrium analysis would provide a new interpretation for the marginal revolution in the history of economics.

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