Hyperbolic Discounting and the Closed-end Fund Puzzle

Takayuki Ogawa
Osaka University of Economics

Hiroaki Ohno
Meiji Gakuin University

Discussion Paper No.24-02
2024 年 12 月

Hyperbolic Discounting and the Closed-end Fund Puzzle*

TAKAYUKI OGAWA[†] HIROAKI OHNO[‡]

Abstract

We show that hyperbolic discounting is key for understanding the closed-end fund puzzle, which is the persistent deviation between the market prices of closed-end funds and net asset value (NAV) of their underlying assets. Hyperbolic-discounting individuals are willing to utilize illiquid underlying assets as commitment devices to avoid time-inconsistent consumption plans. Consequently, more liquid closed-end funds trade at discounts to their NAV. We establish the conditions under which the discounts and premiums arise in the prices of closed-end funds.

Keywords: Closed-end fund puzzle, Commitment, Hyperbolic discounting, Illiquid assets, Short-sale constraint.
JEL Classification Numbers: E21, E70, G12, G51.

Introduction

Arbitrage is one of the most fundamental principles in economics, wherein assets with identical payoff structures are priced the same. This does not necessarily hold for the pricing of closed-end funds (CEFs), which has intrigued researchers as the closed-end fund puzzle. CEFs typically trade at discounts and sometimes at premiums to their net asset value (NAV), which is the per share market value of the underlying assets (UAs). Using an asset pricing model with time inconsistent preferences due to hyperbolic discounting, we show that the illiquidity of UAs, combined with the short-sale constraints on CEFs, creates a discount in the CEF price, thereby serving as a commitment device. By removing future choices, illiquid assets rather help in achieving time-consistent consumption plans. Thus, more liquid CEFs are priced under UAs. We also reveal that the opposite case can occur: When current income is low, and thus, the short-sale constraint is binding, high demand for CEFs results in premiums instead of discounts on the CEF prices. These two imperfections in the financial market are key to unraveling the CEF puzzle.

A CEF, exemplified by real estate investment trusts (REITs), issues a fixed number of shares to raise capital through an initial public offering (IPO). This capital is invested in various UAs, including highly illiquid assets, such as real estate. Once a fund is launched, the shares are traded on stock exchange markets without new issuances and redemptions until maturity. Therefore, the CEF can be interpreted as a way for liquidating otherwise illiquid assets. The market prices of CEFs usually deviate from their

*Takayuki Ogawa is with Osaka University of Economics. Hiroaki Ohno is with Meiji Gakuin University. We are grateful to Kenjiro Hirata, Shiba Suzuki, and the seminar participants at the 6th Fall Conference of the Nippon Finance Association for their helpful comments. We appreciate the financial support from JSPS KAKENHI (grant numbers 19K01767, 20K01669, 20K01778) and the Program of the Joint Usage/Research Center for Behavioral Economics at the Institute of Social and Economic Research (ISER), Osaka University.

[†]Faculty of Economics, Osaka University of Economics, 2-2-8 Osumi, Higashiyodogawa-ku, Osaka, 533-8533, JAPAN. Tel: +81-6-6328-2431. E-mail: tkogawa@osaka-ue.ac.jp.

[‡]Correspondence: Department of Economics, Meiji Gakuin University, 1-2-37 Shirokanedai, Minato-ku, Tokyo, 108-0071, JAPAN. Tel: +81-3-5421-5342. E-mail: ohno@eco.meijigakuin.ac.jp.

NAV. Notably, the CEF price trades at a premium of almost 10% after the IPO and transitions to a discount of over 10% within 120 days on average (Rosenfeldt and Tuttle (1973), Weiss (1989), Lee, Shleifer, and Thaler (1991), Cherkes, Sagi, and Stanton (2009)). This discount persists until the CEF is ultimately converted into an open-end fund (Brauer (1984), Brickley and Schallheim (1985)). Studies attribute the initial premium to underwriting and start-up costs, which are paid out of IPO proceeds (Lee, Shleifer, and Thaler (1991), Cherkes, Sagi, and Stanton (2009)). However, the subsequent discounts are considered as a "free lunch," which refers to a potential profit opportunity. Buying CEFs and selling UAs can yield significantly positive abnormal returns (Thompson (1978), Herzfeld (1980), Richards, Fraser, and Groth (1980), Brauer (1988)). Here, we concentrate on elucidating why CEF prices persistently deviate from their NAV, particularly with discounts, even after a substantial period after the IPO.

According to Cherkes's (2012) survey, many studies have attempted to explain the CEF puzzle from various perspectives, including investor irrationality, management fees associated with asset liquidation, information asymmetry regarding fund managers' ability, tax overhang stemming from unrealized capital gains in CEFs, inaccuracies in reported NAV attributable to bookkeeping complications, disruption of arbitrage due to market imperfections, and CEF premia driven by leverage provisions. Among these, two theories exert a predominant influence over CEF pricing research. The first is De Long et al.'s (1990) investor sentiment theory. The liquidity transformation of illiquid UAs into a CEF, along with no minimum investment requirements, facilitates small investors' participation in capital markets. Given that the behavior of small investors is significantly swayed by irrational sentiments such as optimism and pessimism, the CEF price exhibits greater volatility than its NAV. This generates a discount in the CEF price to compensate for the associated undiversifiable risk premium. Lee, Shleifer, and Thaler (1991) provide empirical evidence supporting the investor sentiment hypothesis. While our model is related to investor irrationality, it differs in that illiquid UAs help prevent future time-inconsistent behavior caused by hyperbolic discounting, thereby clarifying the CEF price discount through an increase in the valuation of UAs.

The second is an influential work by Cherkes, Sagi, and Stanton (2009), who consider the tradeoff between the benefits of investing in liquid CEFs and fund manager fees. The underwriting and startup costs incurred at the IPO erode the NAVs of the UAs, whereas the management fees incurred when buying or selling CEFs decrease the value of CEFs. Depending on the strength of the demand for CEFs as liquid assets and magnitude of management fees, either persistent discounts or premiums arise. Berk and Stanton (2007) find a similar result: a discount exists when the value of the fund manager's ability, instead of the liquidity benefits of CEFs, is less than management fees. We contribute to the literature by highlighting another mechanism for the price discrepancy between UAs and CEFs. As hyperbolic-discounting individuals demand illiquidity for self-control, UAs are priced more than CEFs. Our model also has the potential for explaining the persistent premiums observed in CEF prices.

When current income is sufficiently low, individuals have fewer savings motive and thus the short-sale constraints on liquid CEFs become binding. In this situation, the presence of short-sale constraints keeps the demand for CEF and its price high, indicating the emergence of CEF price premiums. This is important because a portion of CEFs maintains their premiums even while the majority trade at discounts (see Cherkes (2012)).

Hyperbolic discounting is a useful model for describing time-inconsistent preferences, wherein the future selves prefer different plans from those set by the current self (Strotz (1956), Phelps and Pollak (1968), Laibson (1997)). A powerful way to ensure that future selves adhere to a time-consistent plan is to utilize commitment devices (Strotz (1956)). Laibson (1997) considers a hyperbolic-discounting preference to clarify that a combination of illiquid assets and borrowing constraints operates as an effective commitment device. Angeletos et al. (2001) develop a quantitative model of Laibson (1997) to investigate why households hold their wealth in an illiquid form. Commitment rationalizes aspects such as mandatory savings facilitated by social security systems (Schwarz and Sheshinski (2007)), savings floors (Malin (2008)), mortgage loans with down-payments (Ghent (2015), Schlafmann (2021)), and owning durable goods instead of renting (Nocke and Peitz (2003), Kang and Kang (2022)).[1] We leverage the idea of commitment devices to understand the price discrepancies between UAs and CEFs through the lens of liquidity differences. Our findings are associated with those of Nocke and Peitz (2003), who find that a secondary market, such as the used goods market, supplies liquidity, thereby impacting the price of durables in the primary market under a hyperbolic discounting preference.

We construct a simple dynamic model that provides a closed-form solution for asset prices and derive the conditions under which CEFs may either trade at a discount or premium. Individuals live for three periods during which they make consumption and portfolio decisions under hyperbolic discounting. The model incorporates illiquid UAs and short-sale constraints of CEFs as a commitment device to control for the behavior of the future selves. Even if the payoff structures are identical between UAs and CEFs, illiquid UAs are priced at a premium relative to CEFs. The feasibility of such commitment strategies hinges on key economic parameters, such as the initial endowment, e, and degree of hyperbolic discounting, β. A CEF price discount materializes when e is relatively large and β is low. However, when β is high, representing weaker hyperbolic discounting, the commitment is no longer viable and the CEF trades at a price equal to its NAV, as in the case of standard exponential discounting and no management fees. Furthermore, when e is sufficiently low, individuals have greater incentives to sell CEFs. Here, the resulting binding short-sale constraints on CEFs generate a CEF price premium. Our framework can describe various situations in CEF pricing. Commitment is not necessarily be guaranteed even under hyperbolic discounting, although most studies do not focus on this possibility (exceptions include Amador, Werning, and Angeletos (2006), Ambrus and Egorov (2013) and Ogawa and Ohno (2024)).

The remainder of the article proceeds as follows: Section 1 presents the basic model structure. Section 2 derives the optimality conditions for a hyperbolic-discounting individual. Section 3 yields the equilibrium asset prices, and establishes the conditions under which the CEF price discount and premium arise. Finally, section 4 presents the conclusion of the study.

1 The Model

A CEF is an investment scheme that raises a fixed amount of capital through an IPO. In contrast to open-end funds, a CEF does not issue or redeem shares based on investor demand until the initially issued shares reach maturity. The collected funds are invested in equities, bonds, real estate, infrastructure projects, etc.. The shares are traded on stock exchange markets, usually at a premium or discount relative to the UAs' per-share market value, that is, the NAV. The CEF's price deviation from its NAV is known as the asset pricing puzzle. Although some CEFs are traded at premiums, most empirical literature finds CEF price discounts within 120 days, on average, after the IPO (Rosenfeldt and Tuttle (1973), Weiss (1989) Lee, Shleifer, and Thaler (1991), Cherkes, Sagi, and Stanton (2009)). Here, we explore the mechanism via which hyperbolic discounting in preference creates the price deviation between the liquid CEFs and illiquid UAs.

Consider identical individuals who hyperbolically discount their future utility streams. An individual lives for three periods, denoted by 1, 2, and 3, while the economy lasts infinitely. Two types of financial assets exist with different degrees of liquidity. The first is an asset that generates an income gain of $A(> 0)$ in every period, which we call an UA. Individuals can buy or sell it in period 1 but

[1] See Bryan, Karlan, and Nelson (2010) for a survey on commitment.

cannot liquidate it in period 2, requiring them to hold it until period 3. This assumption reflects the fact that the UAs of CEFs contain some illiquid assets such as real estate and infrastructure projects; in other words, a CEF transforms its illiquid UAs into a liquid asset (Cherkes (2012)). For example, REITs provide a way to liquidate investments in illiquid real estate. The second asset is a CEF, which is more liquid than a portfolio of its UAs as it is traded in every period in a stock exchange market. The CEF also yields an income gain of A in each period. However, because of the management fees involved in running the fund, individuals receive dA after deducting these costs, where $0 < d \leq 1$. Following Laibson (1997) and Ogawa and Ohno (2024), we assume that the short selling of liquid CEF is prohibited. Individuals with hyperbolic discounting utilize the liquidity differences between UA and CEF to commit to future time-consistent consumption plans. Thus, the two assets are no longer indifferent and their prices may deviate even without management fees ($d = 1$).

In period 1, an individual receives an endowment $\bar{e}(> 0)$ and divides it into consumption c_1, the illiquid UA b_1^U, and the liquid CEF b_1^C. Thus, the flow budget equation in period 1 is given by:

$$c_1 + p_1^U b_1^U + p_1^C b_1^C = \bar{e}, \tag{1}$$

where p_1^U and p_1^C are the market prices of UA and CEF, respectively. In period 2, the individual receives income gains Ab_1^U and dAb_1^C from the UA and CEF, respectively. While the UA cannot be liquidated, the CEF can be traded at the market price p_2^C in period 2. The remaining funds are directed toward consumption, c_2. Accordingly, the flow budget equation in period 2 satisfies

$$c_2 + p_2^C b_2^C = Ab_1^U + \left(dA + p_2^C\right) b_1^C, \tag{2}$$

where b_2^C denotes the amount of the CEF held at the end of period 2. In period 3, the individual obtains income and capital gains, and allocates them all to consumption c_3:

$$c_3 = \left(A + p_3^U\right) b_1^U + \left(dA + p_3^C\right) b_2^C. \tag{3}$$

Since the UAs and CEF inherently have the same payoff structure, we can interpret p^U as the CEF's NAV. The closed-end fund puzzle represents the deviation of p^C from p^U.

Outside this economy, fully liquid assets are traded at a constant gross interest rate of $\bar{r} > 1$. Hence, the no-arbitrage condition between these assets and CEF implies that:

$$\bar{r} = \frac{dA + p_2^C}{p_1^C} = \frac{dA + p_3^C}{p_2^C} = \frac{dA + p_{t+1}^C}{p_t^C}, \tag{4}$$

where t denotes a time index. The dynamic equation $p_{t+1}^C = \bar{r}p_t^C - dA$ is unstable because the eigenvalue $\bar{r}$ exceeds 1. Hence, the CEF's market price initially jumps to

$$p^C = \frac{dA}{\bar{r} - 1}(> 0) \tag{5}$$

and stays there afterward.[2] As Cherkes, Sagi, and Stanton (2009), the CEF price decreases when management fees are higher, indicating a lower d. Given this price, the CEF is elastically supplied outside the economy. By contrast, the supply of UA is exogenously fixed at $\bar{z} > 0$, thereby fulfilling the following relationship in equilibrium:

$$b_1^U = b_t^U = \bar{z}(> 0). \tag{6}$$

[2]Our model has an overlapping-generations structure. Hence, bubbles may emerge if the economy is dynamically inefficient (Tirole (1985)). In such a case, bubbles in the CEF raise its price beyond its fundamental value, $dA/(\bar{r} - 1)$.

The price of UA is determined by the market, reflecting its demand.

Given the time-inconsistent nature of preferences under hyperbolic discounting, we can describe individual decision-making as a game between the inner *selves* with different preferences (Phelps and Pollak (1968), Pollak (1968), Laibson (1997)). We refer to the self in period t as the *self t*. The utility stream for self 1 is defined by

$$u(c_1) + \beta_1 \delta \left[u(c_2) + \delta u(c_3) \right], \quad \text{with} \quad 0 < \delta < 1 \quad \text{and} \quad 0 < \beta_1 < 1, \tag{7}$$

where δ is the discount factor, β_1 measures self 1's degree of hyperbolic discounting, and $u(c)$ is the felicity function from consumption, which is strictly increasing, strictly concave, continuously differentiable, and satisfies the Inada condition. Self 1 discounts the period-3 utility by δ relative to the period-2 utility. As time passes, self 2 becomes more impatient, discounting the period-3 utility by $\beta_2 \delta$ instead of δ:

$$u(c_2) + \beta_2 \delta u(c_3), \quad \text{with} \quad 0 < \beta_2 < 1, \tag{8}$$

where β_2 represents the self 2's degree of hyperbolic discounting. Throughout this study, we restrict our analysis under the following logarithmic utility function to obtain a closed-form solution for asset prices:[3]

$$u(c) = \ln c.$$

Owing to the change in preference from (7) to (8), the optimal consumption path for self 1 differs from that for self 2. That is, time inconsistency arises in the preference. However, self 1 manages to control for self 2's behavior using the illiquid UA and short-sale constraints on the CEF, which are represented by

$$b_t^C \geq 0. \tag{9}$$

There are several practical reasons to assume short-sale constraints. In most cases, the trading volume of CEFs is low, making it difficult to borrow shares for short selling. This low liquidity may cause large price fluctuations, inducing investors to avoid short selling. Additionally, the financial costs associated with short selling, such as borrowing fees and the obligation to pay dividends to the owner of borrowed shares, are often substantial and may outweigh the benefits of the short sale. Finally, some CEFs are, by regulation, restricted or prohibited from being short sold.

2 Optimization

We consider a subgame-perfect equilibrium in a game played between the selves 1 and 2. Self 1 designs her consumption plan, expecting self 2's reaction. Taking the assets left by self 1 as given, self 2 attempts to revise her consumption plan according to her own preference (8); however, this may not be achievable due to the presence of the illiquid UA and short-sale constraints on the CEF. This section first derives self 2's reaction function and then self 1's optimality conditions. In the next section, we determine the subgame-perfect equilibrium.

[3] Our main implications on asset prices are the same if we employ a more general utility function. See, for example, Ogawa and Ohno (2024).

2.1 Self 2's decision

Given the predetermined amounts of the UA and CEF, self 2 adjusts her CEF holdings in period 2, b_2^C, to maximize her own utility (8), subject to the budget equations (2)–(3) and short-sale constraint (9) wherein $t = 2$:

$$\max_{b_2^C} \ \ln\left[Ab_1^U + \left(dA + p_2^C\right)b_1^C - p_2^C b_2^C\right] + \beta_2\delta \ln\left[\left(A + p_3^U\right)b_1^U + \left(dA + p_3^C\right)b_2^C\right] + \mu_2 b_2^C,$$

where μ_t denotes the Lagrange multiplier for $b_2^C \geq 0$. Considering the no-arbitrage condition (4), we obtain the following first-order optimality condition:

$$p_2^C \geq \frac{\beta_2\delta c_2}{c_3}\left(dA + p_3^C\right), \quad \text{or equally,} \quad \frac{1}{\bar{r}} \geq \frac{\beta_2\delta c_2}{c_3}, \quad \text{with equality when } b_2^C > 0.$$

Since self 2 cannot alter the amount of illiquid UA, she may be unable to intertemporally smooth her consumption due to the short-sale constraint on CEF.

When the short-sale constraint is not binding in period 2 ($b_2^C > 0$), self 2 can flexibly choose the desired consumption plan by adjusting the CEF amount at the end of period 2. Eliminating c_2 and c_3 from self 2's optimality condition with equality using the budget equations (2)–(3) yields her reaction function:

$$\begin{aligned} c_2 &= \frac{1}{1+\beta_2\delta}\left[\left(A + \frac{A + p_3^U}{\bar{r}}\right)b_1^U + \left(dA + p_2^C\right)b_1^C\right], \\ c_3 &= \frac{\beta_2\delta\bar{r}}{1+\beta_2\delta}\left[\left(A + \frac{A + p_3^U}{\bar{r}}\right)b_1^U + \left(dA + p_2^C\right)b_1^C\right], \\ b_2^C &= \frac{1}{\left(1+\beta_2\delta\right)p_2^C}\left\{\beta_2\delta\left[Ab_1^U + \left(dA + p_2^C\right)b_1^C\right] - \frac{A + p_3^U}{\bar{r}}b_1^U\right\} (> 0). \end{aligned} \tag{10}$$

The square bracket in the second and third equations represents the lifetime income in period 2, where b_1^U and b_1^C are predetermined by self 1. Self 2 allocates it to period-2 and period-3 consumptions in the ratio 1 to $\beta_2\delta$ based on her own utility (8). This consumption allocation is possible if the short-sale constraint is not binding. From the third equation in (10), the existing condition for this consumption allocation is

$$\textbf{Condition 1:} \quad b_2^C > 0, \quad \text{or equivalently,} \quad \frac{A + p_3^U}{\bar{r}}b_1^U < \beta_2\delta\left[Ab_1^U + \left(dA + p_2^C\right)b_1^C\right].$$

By contrast, when the short-sale constraint is binding, self 2 has no choice but to choose the consumption allocation that is not optimal for herself. Substituting $b_2^C = 0$ into the budget equations (2)–(3) yields

$$c_2 = Ab_1^U + \left(dA + p_2^C\right)b_1^C, \quad c_3 = \left(A + p_3^U\right)b_1^U, \quad b_2^C = 0. \tag{11}$$

Self 2 cannot smooth her consumption intertemporally, implying that her optimality condition does not necessarily hold with equality:

$$\textbf{Condition 2:} \quad \frac{1}{\bar{r}} \geq \frac{\beta_2\delta c_2}{c_3}, \quad \text{or equivalently,} \quad \beta_2\delta\left[Ab_1^U + \left(dA + p_2^C\right)b_1^C\right] \leq \frac{A + p_3^U}{\bar{r}}b_1^U.$$

The impatient self 2 wants to shift income from period 3 to 2 to increase period-2 consumption, but cannot do so due to the binding short-sale constraint on the CEF. Although this situation is undesirable for self 2, the less impatient self 1 manages to achieve this by adjusting the amount of the illiquid UA.

We summarize self 2's reaction in the following lemma:

Lemma 1

(a) If Condition 1 is satisfied, then the short-sale constraint on the CEF is not binding in period 2 *and self* 2*'s reaction is given by (10), where consumption depends on self* 2*'s degree of hyperbolic discounting,* β_2.

(b) If Condition 2 is satisfied, then the short-sale constraint on the CEF is binding in period 2 *and self* 2*'s reaction is given by (11), where consumption is independent of self* 2*'s degree of hyperbolic discounting,* β_2.

Given that the other variables remain the same, a larger b_1^C implies a lower incentive for self 2 to shift her income from period 3 to 2. Thus, self 2 is not restricted by the short-sale constraint on CEF and flexibly chooses her desirable consumption plan. Meanwhile, with a relatively small b_1^C, self 2 is forced to follow the consumption plan designed by self 1.

2.2 Self 1's decision

We next analyze the self 1's demand for the UA and CEF. There are four kinds of candidates for self 1's strategy, depending on whether the short-sale constraints on the CEF in periods 1 and 2 are binding:

- Case 1a: $b_1^C > 0$ and $b_2^C = 0$.
- Case 1b: $b_1^C = 0$ and $b_2^C = 0$.
- Case 2a: $b_1^C > 0$ and $b_2^C > 0$.
- Case 2b: $b_1^C = 0$ and $b_2^C > 0$.

In Cases 2a and 2b where self 2 can sell the CEF without being constrained, self 2 can flexibly revise the consumption plan made by self 1. That is, self 1 cannot commit to her consumption plan in the future. We refer to these types of strategies as *flexibility strategies*. In contrast, Cases 1a and 1b are situations in which self 1's commitment is possible. We refer to these strategies as *commitment strategies*. Formally, we define the *commitment* as follows:

Definition 1 *Self 1 can make a commitment if her consumption is independent of self 2's degree of hyperbolic discounting* β_2.

As in Lemma 1, the flexibility strategy arises under Condition 1, whereas the commitment strategy is possible under Condition 2. Next, we focus our analysis on realistic cases by imposing

$$\textbf{Assumption 1:} \quad \delta(\bar{r}-1) < 1, \quad \text{which implies} \quad \beta_2\delta(\bar{r}-1) < 1,$$

where $\bar{r} - 1$ denotes the net interest rate on liquid assets.

2.2.1 Commitment

We first analyze self 1's decision in Cases 1a and 1b where she can commit to her future consumption plan. She maximizes her own utility (7) subject to the period-1 flow budget equation (1), period-1 short-sale constraint (9), in which $t = 1$, and self 2's reaction function (11):

$$\max_{b_1^U, b_1^C} \ln\left(\bar{e} - p_1^U b_1^U - p_1^C b_1^C\right) + \beta_1\delta \ln\left[A b_1^U + \left(dA + p_2^C\right) b_1^C\right] + \beta_1\delta^2 \ln\left(A + p_3^U\right) b_1^U + \mu_1 b_1^C,$$

where μ_1 denotes the Lagrange multiplier for $b_1^C \geq 0$. Note that even if the short-sale constraint on the UA is present, self 1 chooses a positive amount of the UA to avoid zero consumption in period 3. The first-order optimality conditions require:

$$p_1^C \geq \frac{\beta_1 \delta c_1}{c_2} \left(dA + p_2^C \right), \text{ or equally, } \frac{1}{\bar{r}} \geq \frac{\beta_1 \delta c_1}{c_2}, \text{ with equality when } b_1^C > 0, \tag{12}$$

$$p_1^U = \frac{\beta_1 \delta c_1}{c_2} \left[A + \frac{\delta c_2}{c_3} \left(A + p_3^U \right) \right], \tag{13}$$

where we use the no-arbitrage condition (4) to obtain the second equation in (12).

Two cases arise, depending on whether self 1 is restricted from selling CEF in period 1: When $b_1^C > 0$, self 1 can commit without being constrained to sell the CEF. Thus, we call this the *unconstrained commitment strategy*. Meanwhile, when $b_1^C = 0$, self 1 is willing to take the negative holding position of CEF but cannot. We refer to this as the *constrained commitment strategy*.

Case 1a: Unconstrained commitment

Consider the case in which $b_1^C > 0$ and $b_2^C = 0$. Because self 1's optimality condition (12) holds with equality, we have $\beta_1 \delta \bar{r} c_1 = c_2$, where c_1 and c_2 satisfy

$$c_1 = \bar{e} - p_1^U b_1^U - p_1^C b_1^C, \quad c_2 = A b_1^U + \bar{r} p_1^C b_1^C,$$

from the period-1 flow-budget equation (1) and self 2's reaction function (11), to which the no-arbitrage condition (4) is applied, respectively. This solves self 1's CEF demand:

$$p_1^C b_1^C = \frac{\beta_1 \delta}{1 + \beta_1 \delta} \left(\bar{e} - p_1^U b_1^U \right) - \frac{A}{(1 + \beta_1 \delta)\, \bar{r}} b_1^U, \tag{14}$$

which rewrites c_1 and c_2 as

$$c_1 = \frac{1}{1 + \beta_1 \delta} \left[\bar{e} - \left(p_1^U - \frac{A}{\bar{r}} \right) b_1^U \right], \quad c_2 = \frac{\beta_1 \delta \bar{r}}{1 + \beta_1 \delta} \left[\bar{e} - \left(p_1^U - \frac{A}{\bar{r}} \right) b_1^U \right].$$

We eliminate c_1, c_2, and c_3 from self 1's optimality condition (13) using the above consumption equations and second equation in the self 2's reaction function (11). We then apply the equilibrium condition in the UA market (6) to the result. This provides the equilibrium price of the UA:

$$p^U = \frac{A}{\bar{r}} + \frac{\beta_1 \delta^2 \bar{e}}{\left(1 + \beta_1 \delta + \beta_1 \delta^2\right) \bar{z}}, \tag{15}$$

which is constant over time. Thus, the equilibrium consumption allocation is given by:

$$c_1 = \frac{\bar{e}}{1 + \beta_1 \delta + \beta_1 \delta^2}, \quad c_2 = \frac{\beta_1 \delta \bar{r} \bar{e}}{1 + \beta_1 \delta + \beta_1 \delta^2}, \quad c_3 = \frac{\beta_1 \delta^2 \bar{r} R \bar{e}}{1 + \beta_1 \delta + \beta_1 \delta^2},$$

where

$$R \equiv \frac{c_3}{\delta c_2} = \frac{1}{\bar{r}} + \frac{\left(1 + \beta_1 \delta + \beta_1 \delta^2\right)(1 + \bar{r}) A \bar{z}}{\beta_1 \delta^2 \bar{r}^2 \bar{e}}. \tag{16}$$

With this unconstrained commitment strategy, self 1 can allocate the initial endowment $\bar{e}$ to consumption in periods 1, 2, and 3 according to her own preference (7) in the proportions 1, $\beta_1 \delta$, and $\beta_1 \delta^2$, respectively. R is interpreted as the implied gross interest rate based on self 1's marginal rate of substitution of period 2-consumption for period-3 consumption.

Substituting the equilibrium asset prices (5) and (15), and the equilibrium condition in the UA market (6) into (14) yields

$$b_1^C = \frac{\beta_1\delta\left(\bar{r}-1\right)}{\left(1+\beta_1\delta+\beta_1\delta^2\right)dA}\left[\bar{e}-\frac{\left(1+\beta_1\delta+\beta_1\delta^2\right)A\bar{z}}{\beta_1\delta\bar{r}}\right],$$

which must be positive under the unconstrained commitment strategy:

$$b_1^C > 0, \quad \text{or equivalently,} \quad \bar{e} > \frac{\bar{r}-1}{\bar{r}}\Omega(>0), \quad \text{where} \quad \Omega \equiv \frac{\left(1+\beta_1\delta+\beta_1\delta^2\right)A\bar{z}}{\beta_1\delta\left(\bar{r}-1\right)}(>0). \tag{17}$$

The equilibrium consumption allocation rewrites Condition 2 for $b_2^C = 0$ as $\left(\beta_2\bar{r}^2-1\right)\bar{e} \leq \frac{(1+\bar{r})(\bar{r}-1)}{\delta\bar{r}}\Omega$. As $\bar{e} > 0$, we can reduce this condition to:

$$\begin{cases} 0 < \bar{e} \leq \frac{(1+\bar{r})(\bar{r}-1)}{(\beta_2\bar{r}^2-1)\delta\bar{r}}\Omega & \text{if} \quad \beta_2\bar{r}^2 > 1; \\ 0 < \bar{e} & \text{if} \quad \beta_2\bar{r}^2 \leq 1. \end{cases}$$

Appendix A shows that

$$\frac{\bar{r}-1}{\bar{r}}\Omega < \frac{\left(1+\bar{r}\right)\left(\bar{r}-1\right)}{\left(\beta_2\bar{r}^2-1\right)\delta\bar{r}}\Omega \quad \text{under } \beta_2\bar{r}^2 > 1 \text{ and Assumption 1.}$$

Therefore, the necessary condition for the existence of the unconstrained commitment equilibrium is summarized as follows:

Lemma 2 *Under Assumption 1, the unconstrained commitment equilibrium with $b_1^C > 0$ and $b_2^C = 0$ exists when:*

$$\begin{cases} \frac{\bar{r}-1}{\bar{r}}\Omega < \bar{e} \leq \frac{(1+\bar{r})(\bar{r}-1)}{(\beta_2\bar{r}^2-1)\delta\bar{r}}\Omega & \text{if} \quad \beta_2\bar{r}^2 > 1; \\ \frac{\bar{r}-1}{\bar{r}}\Omega < \bar{e} & \text{if} \quad \beta_2\bar{r}^2 \leq 1; \end{cases}$$

where Ω is expressed by (17).

Next, we investigate the implications for asset pricing. By combining (13) and the second equation in (12), in which the equality holds, and using R in (16) to eliminate $\delta c_2/c_3$ from the result, we have:

$$p^U = \frac{A}{\bar{r}} + \frac{A+p^U}{\bar{r}R}, \quad \text{or equally,} \quad p^U = \frac{\left(1+R\right)A}{\bar{r}R-1}(>0),$$

both of which equal (15) under (16). The first equation states that the price of UA equals the present value of the future UA's payoffs. Notably, the period-3 payoff is discounted by $\bar{r}R$, not by $\bar{r}^2$, based on self 1's preference for consumption in periods 2 and 3. Dividing the second equation by the CEF price (5) yields

$$\frac{p^U}{p^C} = \frac{\left(\bar{r}-1\right)\left(1+R\right)}{\left(\bar{r}R-1\right)d} \gtreqless 1 \quad \text{if} \quad \bar{r}+\left(\bar{r}R-1\right)\left(1-d\right) \gtreqless R.$$

Because R in (16) is independent of d, a higher management fee—or a smaller d—indicates a larger discount on the CEF price relative to its NAV ($p^C < p^U$). Management fees create the discount even when $R = \bar{r}$. To explain the discounts on CEF prices, Cherkes, Sagi, and Stanton (2009) emphasize the role of management fees involved when CEFs are liquidated.

In our model with hyperbolic discounting, the discount can arise through another channel. To observe this, we assume away management fees ($d = 1$). Considering Condition 2, we find that $p^C < p^U$ holds if

$$\frac{1}{R} = \frac{\delta c_2}{c_3} > \frac{1}{\bar{r}} \geq \frac{\beta_2 \delta c_2}{c_3}.$$

Self 1 is less impatient to consume in period 2 than self 2 is. This yields a lower implied interest rate R between periods 2 and 3, indicating a higher present value of the period-3 payoff, and thus, higher price of UA. That is, the UA has extra value as a device for achieving the commitment in the presence of time-inconsistent preferences. While management fees decrease the CEF price, hyperbolic discounting resolves this puzzle by increasing the UA price. Note that the range with the discount disappears if the preference is time-consistent; that is, if $\beta_2 = 1$.

Using R in (16), into which the equilibrium consumption allocation is substituted, we obtain the following proposition:

Proposition 1 *Suppose there are no management fees ($d = 1$). Then, the following equilibrium relationship for asset prices holds in the unconstrained commitment equilibrium:*

$$p^U \gtreqless p^C, \quad \textit{or equally,} \quad \bar{r} \gtreqless R, \quad \textit{if} \quad \bar{e} \gtreqless \frac{1}{\delta \bar{r}} \Omega,$$

where Ω is expressed by (17).

A smaller $\bar{z}$, which implies a smaller Ω, lowers consumption in period 3. This yields a lower R, thereby generating a discount on the CEF price. However, note that the commitment strategy does not always guarantee a discount; a premium on the CEF price may arise for a larger $\bar{z}$. In section 3, we consider all possible strategies for self 1 and clarify the conditions under which the CEF price deviates from the UA price, either at a discount or premium.

Case 1b: Constrained commitment

Consider the case where $b_1^C = b_2^C = 0$. Substituting $b_1^C = 0$ into the period-1 flow budget equation (1) and self 2's reaction function (11) yields:

$$c_1 = \bar{e} - p_1^U b_1^U, \quad c_2 = A b_1^U, \quad c_3 = \left(A + p_3^U\right) b_1^U.$$

Applying these consumption functions and the UA's equilibrium condition (6) to self 1's optimality condition (13), we obtain the following equilibrium price for the UA:

$$p^U = \frac{\left(\beta_1 \delta + \beta_1 \delta^2\right) \bar{e}}{\left(1 + \beta_1 \delta + \beta_1 \delta^2\right) \bar{z}}, \tag{18}$$

which gives the equilibrium consumption allocation as follows:

$$c_1 = \frac{\bar{e}}{1 + \beta_1 \delta + \beta_1 \delta^2}, \quad c_2 = A\bar{z}, \quad c_3 = A\bar{z} + \frac{\left(\beta_1 \delta + \beta_1 \delta^2\right) \bar{e}}{1 + \beta_1 \delta + \beta_1 \delta^2}.$$

The short-sale constraint on the CEF prevents self 1 from intertemporally smoothing their consumption, although the commitment remains possible. Owing to the binding short-sale constraint on the CEF ($b_1^C = 0$), self 1's optimality condition (12) does not necessarily hold with equality. The equilibrium consumption allocation reduces (12) and Condition 2 to:

$$0 < \bar{e} \leq \frac{\bar{r} - 1}{\bar{r}} \Omega, \qquad \begin{cases} \bar{e} \geq \frac{(\beta_2 \delta \bar{r} - 1)(\bar{r} - 1)}{1 + \delta} \Omega & \text{if} \quad \beta_2 \delta \bar{r} > 1; \\ \bar{e} > 0 & \text{if} \quad \beta_2 \delta \bar{r} \leq 1, \end{cases}$$

where Ω is defined by (17). As formally proven in Appendix A, Assumption 1 guarantees that:

$$\frac{(\beta_2\delta\bar{r}-1)(\bar{r}-1)}{1+\delta}\Omega < \frac{\bar{r}-1}{\bar{r}}\Omega.$$

Therefore, the existence condition of the constrained commitment equilibrium can be summarized as follows:

Lemma 3 *Under Assumption 1, the constrained commitment equilibrium with $b_1^C = b_2^C = 0$ can exist when:*

$$\begin{cases} \frac{(\beta_2\delta\bar{r}-1)(\bar{r}-1)}{1+\delta}\Omega \leq \bar{e} \leq \frac{\bar{r}-1}{\bar{r}}\Omega & \text{if} \quad \beta_2\delta\bar{r} > 1; \\ 0 < \bar{e} \leq \frac{\bar{r}-1}{\bar{r}}\Omega & \text{if} \quad \beta_2\delta\bar{r} \leq 1; \end{cases}$$

where Ω is expressed by (17).

We can understand the implications of the asset prices by dividing the UA price (18) by the CEF price (5):

$$\frac{p^U}{p^C} = \frac{\left(\beta_1\delta+\beta_1\delta^2\right)(\bar{r}-1)\bar{e}}{(1+\beta_1\delta+\beta_1\delta^2)\,dA\bar{z}} \gtreqless 1 \quad \text{if} \quad \bar{e} \gtreqless \frac{d}{1+\delta}\Omega.$$

Without management fees ($d = 1$), we cannot explain the discount on the CEF price within the existence condition presented in Lemma 3 because

$$\frac{1}{1+\delta}\Omega - \frac{\bar{r}-1}{\bar{r}}\Omega = \frac{1-\delta(\bar{r}-1)}{(1+\delta)\,\bar{r}} > 0$$

under Assumption 1. The presence of short-sale constraints keeps the demand for CEF and its price high. Within the condition in Lemma 3, this effect outweighs the upward pressure on the UA price, driven by the increased demand for the UA as a commitment device. Consequently, we find that:

Proposition 2 *Suppose there are no management fees ($d = 1$). The CEF is traded at a premium, not a discount, in the constrained commitment equilibrium: $p^U < p^C$.*

2.2.2 Flexibility

Outside the region presented in Lemmas 2 and 3 where commitment is possible, self 1 cannot control for self 2's behavior. In other words, self 2 has the flexibility to make consumption decisions in periods 2 and 3. In this situation, self 1 maximizes her own utility (7) subject to the period-1 flow budget constraint (1), period-1 short-sale constraint (9), in which $t = 1$, and self 2's reaction function (10):

$$\begin{aligned}\max_{b_1^U, b_1^C} \ & \ln\left(\bar{e} - p_1^U b_1^U - p_1^C b_1^C\right) + \beta_1\delta \ln \frac{1}{1+\beta_2\delta}\left[\left(A + \frac{A+p_3^U}{\bar{r}}\right)b_1^U + \left(dA + p_2^C\right)b_1^C\right] \\ & + \beta_1\delta^2 \ln \frac{\beta_2\delta\bar{r}}{1+\beta_2\delta}\left[\left(A + \frac{A+p_3^U}{\bar{r}}\right)b_1^U + \left(dA + p_2^C\right)b_1^C\right] + \mu_1 b_1^C,\end{aligned}$$

where μ_1 is the Lagrange multiplier for $b_1^C \geq 0$.

Considering that $c_3 = \beta_2\delta\bar{r}c_2$ is satisfied from (10) due to self 2's flexible decision, we have the following first-order optimality conditions:

$$p_1^C \geq \frac{\left(\beta_1\delta+\beta_1\delta^2\right)c_1}{(1+\beta_2\delta)\,c_2}\left(dA + p_2^C\right), \text{ or equally, } \frac{1}{\bar{r}} \geq \frac{\left(\beta_1\delta+\beta_1\delta^2\right)c_1}{(1+\beta_2\delta)\,c_2}, \text{ with equality when } b_1^C > 0, \tag{19}$$

$$p_1^U = \frac{\left(\beta_1\delta+\beta_1\delta^2\right)c_1}{(1+\beta_2\delta)\,c_2}\left(A + \frac{A+p_3^U}{\bar{r}}\right), \tag{20}$$

where we used the no-arbitrage condition (4) to obtain the second condition in (12). We refer to the strategy with $b_1^C > (=)0$ as the *unconstrained (constrained) flexibility strategy.*

Case 2a: Unconstrained flexibility

When the short-sale constraints on the CEF are not binding in periods 1 and 2, the following no-arbitrage condition is fulfilled:

$$\bar{r} = \frac{A + \frac{A+p_3^U}{\bar{r}}}{p_1^U} = \frac{A + \frac{A+p_{t+2}^U}{\bar{r}}}{p_t^U}$$

from (19) with equality and (20). Since these price dynamics have two unstable eigenvalues, $\bar{r}(>1)$ and $-\bar{r}(<-1)$, the UA price initially jumps to the steady-state level,

$$p^U = \frac{A}{\bar{r}-1}. \tag{21}$$

Comparing this with the CEF price (5), we find that the CEF is traded at a discount in the presence of management fees, as demonstrated by Cherkes, Sagi, and Stanton (2009):

$$p^C < (=) p^U \quad \text{if} \quad d < (=)1.$$

We summarize this result in the following proposition:

Proposition 3 *Suppose there are no management fees ($d = 1$). The CEF is traded at the same price as the UA in the constrained commitment equilibrium: $p^U = p^C$.*

Next, we derive the equilibrium consumption allocation. Applying the equilibrium asset prices (5) and (21) and the UA's equilibrium condition (6) to the period-1 flow budget constraint (1) and first equation in self 2's reaction function (10), we obtain

$$c_1 = \bar{e} - \frac{A}{\bar{r}-1}\left(\bar{z} + db_1^C\right), \quad c_2 = \frac{\bar{r}A}{(1+\beta_2\delta)(\bar{r}-1)}\left(\bar{z} + db_1^C\right).$$

Substituting these consumption functions into self 1's optimality condition (19) with equality, we obtain the following demand for the CEF:

$$b_1^C = \frac{\left(\beta_1\delta + \beta_1\delta^2\right)(\bar{r}-1)}{\left(1+\beta_1\delta+\beta_1\delta^2\right)dA}\left[\bar{e} - \frac{\left(1+\beta_1\delta+\beta_1\delta^2\right)A\bar{z}}{\left(\beta_1\delta+\beta_1\delta^2\right)(\bar{r}-1)}\right] > 0 \quad \text{if} \quad \bar{e} > \frac{1}{1+\delta}\Omega(>0).$$

This determines the equilibrium consumption allocation as:

$$c_1 = \frac{\bar{e}}{1+\beta_1\delta+\beta_1\delta^2}, \quad c_2 = \frac{\left(\beta_1\delta+\beta_1\delta^2\right)\bar{r}\bar{e}}{\left(1+\beta_1\delta+\beta_1\delta^2\right)(1+\beta_2\delta)}, \quad c_3 = \frac{\left(\beta_1\delta+\beta_1\delta^2\right)\beta_2\delta\bar{r}^2\bar{e}}{\left(1+\beta_1\delta+\beta_1\delta^2\right)(1+\beta_2\delta)}.$$

In period 1, self 1 allocates the initial endowment $\bar{e}$ to period-1 consumption and savings in the proportions 1 and $\beta_1\delta + \beta_1\delta^2$, respectively. Given this decision of self 1, self 2 divides savings into consumption in periods 2 and 3 according to the ratio 1 to $\beta_2\delta$.

The CEF demand in period 2 fulfills the third equation in the self 2's reaction function (10):

$$b_2^C = \frac{\left(\beta_1\delta+\beta_1\delta^2\right)\beta_2\delta\bar{r}(\bar{r}-1)}{\left(1+\beta_1\delta+\beta_1\delta^2\right)(1+\beta_2\delta)dA}\left[\bar{e} - \frac{\left(1+\beta_1\delta+\beta_1\delta^2\right)(1+\beta_2\delta)A\bar{z}}{\left(\beta_1\delta+\beta_1\delta^2\right)\beta_2\delta\bar{r}(\bar{r}-1)}\right] > 0 \quad \text{if} \quad \bar{e} > \frac{1+\beta_2\delta}{\left(\beta_2\delta+\beta_2\delta^2\right)\bar{r}}\Omega.$$

Since it holds that

$$\frac{1+\beta_2\delta}{\left(\beta_2\delta+\beta_2\delta^2\right)\bar{r}}\Omega - \frac{1}{1+\delta}\Omega = \frac{1-\beta_2\delta(\bar{r}-1)}{\left(\beta_2\delta+\beta_2\delta^2\right)\bar{r}}\Omega > 0$$

under Assumption 1, we can summarize the result as follows:

Lemma 4 *Under Assumption 1, the unconstrained flexibility equilibrium with $b_1^C > 0$ and $b_2^C > 0$ can exist when:*

$$\frac{1+\beta_2\delta}{(\beta_2\delta+\beta_2\delta^2)\,\bar{r}}\Omega < \bar{e},$$

where Ω is as defined by (17).

Case 2b: Constrained flexibility

The last case is where $b_1^C = 0$ and $b_2^C > 0$. Substituting $b_1^C = 0$ and the UA's equilibrium condition (6) into the period-1 flow budget constraint (1) and first equation in self 2's reaction function (10), we get

$$c_1 = \bar{e} - p_1^U \bar{z}, \quad c_2 = \frac{\bar{z}}{1+\beta_2\delta}\left(A + \frac{A+p_3^U}{\bar{r}}\right),$$

which solves the equilibrium price of the UA from the self 1's optimality condition (20):

$$p^U = \frac{\left(\beta_1\delta+\beta_1\delta^2\right)\bar{e}}{(1+\beta_1\delta+\beta_1\delta^2)\,\bar{z}}. \tag{22}$$

Accordingly, the equilibrium consumption allocation is:

$$\begin{aligned}
c_1 &= \frac{\bar{e}}{1+\beta_1\delta+\beta_1\delta^2}, \\
c_2 &= \frac{\beta_1\delta+\beta_1\delta^2}{(1+\beta_1\delta+\beta_1\delta^2)\,(1+\beta_2\delta)\,\bar{r}}\left[\bar{e} + \frac{\left(1+\beta_1\delta+\beta_1\delta^2\right)(1+\bar{r})\,A\bar{z}}{\beta_1\delta+\beta_1\delta^2}\right], \\
c_3 &= \frac{\left(\beta_1\delta+\beta_1\delta^2\right)\beta_2\delta}{(1+\beta_1\delta+\beta_1\delta^2)\,(1+\beta_2\delta)}\left[\bar{e} + \frac{\left(1+\beta_1\delta+\beta_1\delta^2\right)(1+\bar{r})\,A\bar{z}}{\beta_1\delta+\beta_1\delta^2}\right],
\end{aligned}$$

where the last equation is derived from the second equation in self 2's reaction function (10). This strategy is feasible if self 1's optimality condition (19) is satisfied:

$$0 < \bar{e} \le \frac{1}{1+\delta}\Omega.$$

The demand for CEF in period 2 fulfills the third equation in self 2's reaction function (10) under the equilibrium asset prices (5) and (22), and UA's equilibrium condition (6):

$$b_2^C = \frac{\left(\beta_1\delta+\beta_1\delta^2\right)(\bar{r}-1)}{(1+\beta_1\delta+\beta_1\delta^2)\,(1+\beta_2\delta)\,\bar{r}dA}\left[\frac{(\beta_2\delta\bar{r}-1)\left(1+\beta_1\delta+\beta_1\delta^2\right)A\bar{z}}{\beta_1\delta+\beta_1\delta^2} - \bar{e}\right],$$

which has a positive sign when

$$\beta_2\delta\bar{r} > 1 \quad \text{and} \quad 0 < \bar{e} < \frac{(\beta_2\delta\bar{r}-1)\,(\bar{r}-1)}{1+\delta}\Omega.$$

Assumption 1 guarantees that:

$$\frac{1}{1+\delta}\Omega - \frac{(\beta_2\delta\bar{r}-1)\,(\bar{r}-1)}{1+\delta}\Omega = \frac{[1-\beta_2\delta\,(\bar{r}-1)]\,\bar{r}}{1+\delta}\Omega > 0.$$

Consequently, we establish the existence condition for the constrained flexibility equilibrium as follows:

Lemma 5 *Under Assumption 1, the constrained flexibility equilibrium with* $b_1^C = 0$ *and* $b_2^C > 0$ *can exist when:*

$$\beta_2 \delta \bar{r} > 1 \quad \textit{and} \quad 0 < \bar{e} < \frac{(\beta_2 \delta \bar{r} - 1)(\bar{r} - 1)}{1 + \delta} \Omega,$$

where Ω *is given by (17).*

The equilibrium asset prices (5) and (22) indicate that

$$\frac{p^U}{p^C} = \frac{\left(\beta_1 \delta + \beta_1 \delta^2\right)(\bar{r} - 1)\bar{e}}{(1 + \beta_1 \delta + \beta_1 \delta^2) \, d A \bar{z}} \gtreqless 1 \quad \text{if} \quad \bar{e} \gtreqless \frac{d}{1 + \delta} \Omega.$$

This relationship is the same as that in Case 1b, where $b_1^C = 0$ and $b_2^C = 0$. Without management fees ($d = 1$), the discount on the CEF price vanishes within the existence condition presented in Lemma 5.

Proposition 4 *Suppose there are no management fees* ($d = 1$)*. The CEF is traded at a premium and not a discount in the constrained flexibility equilibrium:* $p^U < p^C$*.*

Together with Proposition 2, the binding short-sale constraint in period 1 drives the UA price below the CEF price, irrespective of whether the commitment is achievable.

3 Understanding the Closed-End Fund Puzzle

3.1 Equilibrium and Asset Prices

Lemmas 2–5 provide the conditions under which each strategy of self 1 is possible. However, we need to clarify which type of strategy is chosen in terms of self 1's welfare when the existence conditions overlap. Appendix A proves the following relationship between the threshold values presented in Lemma 2–5 and Proposition 1:

$$\frac{(\beta_2 \delta \bar{r} - 1)(\bar{r} - 1)}{1 + \delta} \Omega < \frac{\bar{r} - 1}{\bar{r}} \Omega < \frac{1}{\delta \bar{r}} \Omega < \frac{1 + \beta_2 \delta}{(\beta_2 \delta + \beta_2 \delta^2)\bar{r}} \Omega. \tag{23}$$

Moreover, when $\beta_2 \bar{r}^2 > 1$, we establish the following relationship:

$$\frac{1 + \beta_2 \delta}{(\beta_2 \delta + \beta_2 \delta^2)\bar{r}} \Omega < \frac{(1 + \bar{r})(\bar{r} - 1)}{(\beta_2 \bar{r}^2 - 1)\delta \bar{r}} \Omega. \tag{24}$$

We obtain four cases depending on the signs of $\beta_2 \bar{r}^2 - 1$ and $\beta_2 \delta \bar{r} - 1$. Figures 1 and 2 illustrate the equilibrium choice of self 1 when $\beta_2 \bar{r}^2 > 1$; that is, when the interest rate on liquid assets is relatively high. Within a region of $\frac{1+\beta_2\delta}{(\beta_2\delta+\beta_2\delta^2)\bar{r}}\Omega < \bar{e} \le \frac{(1+\bar{r})(\bar{r}-1)}{(\beta_2\bar{r}^2-1)\delta\bar{r}}\Omega$, both unconstrained commitment and unconstrained flexibility are possible. However, self 1 chooses the former that brings the higher welfare. In Figure 1, where $\beta_2 \delta \bar{r} > 1$, all four types of strategies are adopted as per the level of endowment $\bar{e}$. In Figure 2, where $\beta_2 \delta \bar{r} \le 1$, the region employing the constrained flexibility strategy vanishes. The relationship between the UA and CEF prices when $d = 1$ is also illustrated. Considering Propositions 1–4, we conclude:

Proposition 5 *Suppose there are no management fees* ($d = 1$)*. If* $\beta_2 \bar{r}^2 > 1$*, then asset prices satisfy:*

$$\begin{cases} p^C > p^U & \textit{if} \quad 0 < \bar{e} < \frac{1}{\delta \bar{r}}\Omega; \\ p^C < p^U & \textit{if} \quad \frac{1}{\delta \bar{r}}\Omega < \bar{e} \le \frac{(1+\bar{r})(\bar{r}-1)}{(\beta_2\bar{r}^2-1)\delta\bar{r}}\Omega; \\ p^C = p^U & \textit{if} \quad \bar{e} = \frac{1}{\delta\bar{r}}\Omega \quad \textit{and} \quad \frac{(1+\bar{r})(\bar{r}-1)}{(\beta_2\bar{r}^2-1)\delta\bar{r}}\Omega < \bar{e}. \end{cases}$$

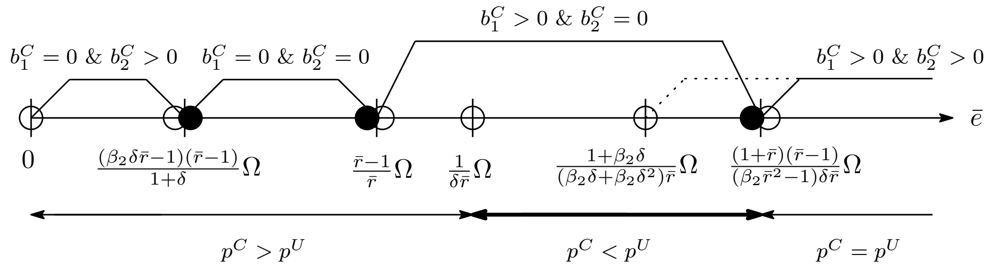

Figure 1: Equilibrium when $\beta_2\bar{r}^2 > 1$ and $\beta_2\delta\bar{r} > 1$.

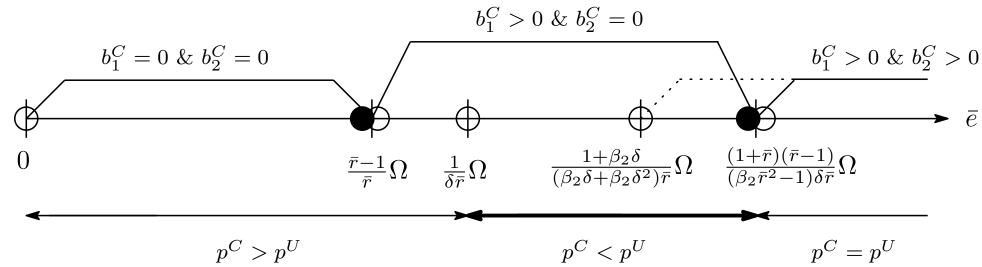

Figure 2: Equilibrium when $\beta_2\bar{r}^2 > 1$ and $\beta_2\delta\bar{r} \leq 1$.

For a low initial endowment $\bar{e}$, self 1 wants to shift future income to period 1; hence, the liquid CEF is traded at a premium to the NAV ($p^C > p^U$). As $\bar{e}$ grows, self 1 becomes capable of making the commitment, which relatively raises the value of the illiquid UA and generates a discount on the CEF price to the NAV ($p^C < p^U$). This discount region does not exist under time-consistent preferences, noting that the following relationship is satisfied:

$$\frac{1}{\delta\bar{r}}\Omega = \frac{1+\beta_2\delta}{(\beta_2\delta+\beta_2\delta^2)\,\bar{r}}\Omega = \frac{(1+\bar{r})\,(\bar{r}-1)}{(\beta_2\bar{r}^2-1)\,\delta\bar{r}}\Omega \quad \text{when} \quad \beta_2 = 1.$$

With a further increase in $\bar{e}$, self 1 cannot control for self 2's behavior using illiquid UA as a commitment device when the interest rate on liquidity assets is sufficiently high to satisfy $\beta_2\bar{r}^2 > 1$. The UA price equals the NAV ($p^C = p^U$) in the absence of management fees ($d = 1$).

Figures 3 and 4 describe the equilibrium when $\beta_2\bar{r}^2 \leq 1$; that is, when self 2's degree of hyperbolic discounting is high. Figure 3 holds when $\beta_2\delta\bar{r} > 1$, whereas Figure 4 holds when $\beta_2\delta\bar{r} \leq 1$. Unlike in Figures 1 and 2, self 1 has a strong incentive to make the commitment and an unconstrained flexibility equilibrium does not appear even for large values of $\bar{e}$. As the endowment $\bar{e}$ grows, the premium on the CEF price shifts to a discount.[4] Accordingly, we can conclude that:

Proposition 6 *Suppose there are no management fees ($d = 1$). If $\beta_2\bar{r}^2 \leq 1$, then asset prices satisfy:*

$$\begin{cases} p^C > p^U & \text{if} \quad 0 < \bar{e} < \frac{1}{\delta\bar{r}}\Omega; \\ p^C < p^U & \text{if} \quad \frac{1}{\delta\bar{r}}\Omega < \bar{e}; \\ p^C = p^U & \text{if} \quad \bar{e} = \frac{1}{\delta\bar{r}}\Omega. \end{cases}$$

[4] We can alternatively interpret the results in terms of the supply of the UA, $\bar{z}$. For a given initial endowment, $\bar{e}$, an increase in $\bar{z}$ increases Ω in (17). This strengthens self 1's incentive to short sell the CEF in period 1, thereby widening the region of the CEF price premium, $p^C > p^U$, as shown in Figures 1–4.

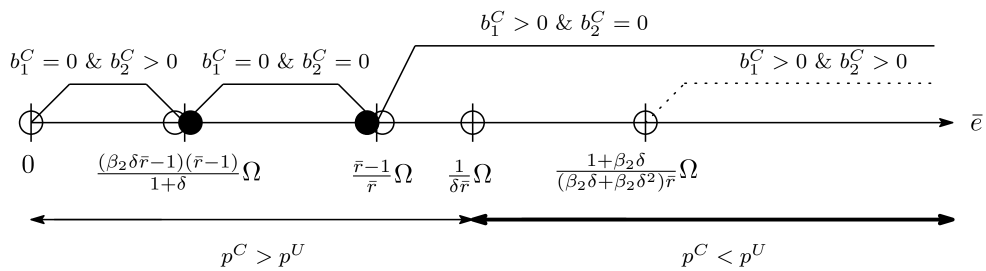

Figure 3: Equilibrium when $\beta_2\bar{r}^2 \leq 1$ and $\beta_2\delta\bar{r} > 1$.

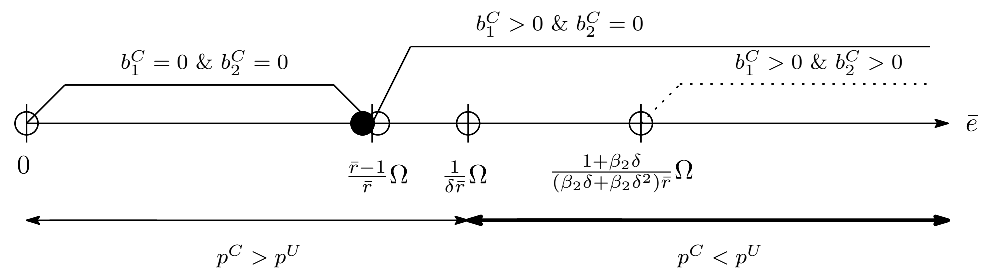

Figure 4: Equilibrium when $\beta_2\bar{r}^2 \leq 1$ and $\beta_2\delta\bar{r} \leq 1$.

Under a time-consistent preference ($\beta_2 = 1$), this case is excluded because the gross interest rate $\bar{r}$ exceeds one, implying that $\beta_2\bar{r}^2 > 1$.

3.2 Discussion

A. Time inconsistency

With time-consistent preferences, such as the standard exponential discounting ($\beta_1 = \beta_2 = 1$) and no management fees ($d = 1$), one cannot explain the CEF price discount. Even if the payoff structures of the UA and CEF are identical, the liquidity difference between these assets can lead to the price discrepancy under hyperbolic discounting, as the illiquid UA serves as a commitment device. This CEF price discount disappears in a region where the commitment is infeasible. Cherkes, Sagi, and Stanton (2009) highlight the importance of the management fees incurred in running CEFs. However, relative to the benefits from CEFs' liquidity services, as a crucial factor in unraveling the CEF discount puzzle, we offer another perspective on the impact of (il)liquidity. Nonetheless, our contribution should be viewed as a complement to Cherkes, Sagi, and Stanton (2009), rather than as a substitute.

Our approach is also closely related to the investor sentiment theory developed by De Long et al. (1990) and empirically tested by Lee, Shleifer, and Thaler (1991). CEFs offer liquidity and require no minimum investments, making them attractive to small investors. Both sets of authors highlight that small investors, swayed by the irrational sentiments of optimism and pessimism, cause substantial undiversifiable fluctuations in CEF prices, leading to price discounts. While they find a mechanism of CEF price declines driven by risk, our model shows UA price appreciation through an incentive to avoid irrational future decisions. One advantage of our approach is that it explains the premiums and discounts in a unified framework. Although the discounts prevail, a measurable portion of the CEFs trade at premiums in actual markets (see Cherkes (2012)).

B. Expectation biases

Earlier, we implicitly posit that self 1 can accurately predict self 2's degree of hyperbolic discounting, β_2. However, following O'Donoghue and Rabin (1999, 2001), we can introduce self 1's expectation bias concerning self 2's preference. We define the true preference for self 2 as follows:

$$u(c_2) + \bar{\beta}_2 \delta u(c_3), \qquad \text{with} \quad 0 < \bar{\beta} < 1,$$

while considering $\beta_2 \in [\bar{\beta}_2, 1]$ as self 2's degree of hyperbolic discounting that self 1 subjectively expects. As per the value of β_2, the degree of self 1's naïveté can be divided into the following three cases:

- With $\beta_2 = \bar{\beta}_2$, self 1 is sophisticated in the sense that she can accurately predict the changes in the discounting held by self 2.
- With $\beta_2 = 1$, self 1 is naïve in the sense that she erroneously believes that self 2 will have the same discounting as her own δ.
- With $\bar{\beta}_2 < \beta_2 < 1$, self 1 is said to be partially naïve.

As self 1 underestimates self 2's degree of hyperbolic discounting, implying a higher β_2 for a given self 2's true preference $\bar{\beta}_2$, the economy shifts to the cases described in Figures 1–2 and further to the region within which $p^C < p^U$ narrows. This indicates that the degree of naïveté also influences asset prices by changing the incentive of self 1 to utilize the illiquid UA as a commitment device.

C. Short-sale constraints

In our setting, the short-sale constraints serve two distinct roles. The future constraint in period 2 prevents the time-inconsistent behavior of the future selves, thus creating a CEF price discount. That is, self 1 achieves the optimal consumption plan with the aid of market imperfections. By contrast, the current constraint in period 1 generates the CEF price premium, distorting self 1's consumption schedule. Without these two constraints, the unconstrained flexibility is the only possible strategy. Further, without management fees, no price discrepancy occurs between the UA and CEF.

Our findings align with the literature on market imperfections impeding arbitrage opportunities as a fundamental source of the CEF puzzle. Miller (1977) posits that short-sale constraints result in financial asset overvaluation by blocking the incorporation of pessimistic evaluations into market prices.[5] Diamond and Verrecchia (1987) clarify that restrictions on short selling diminish the speed at which stock prices reflect private information, particularly bad news.[6] In the context of CEF pricing, Pontiff (1996) demonstrates that arbitrage costs, such as transaction costs, hinder the correction of mispricing in foreign CEFs. Additionally, the author points out that particularly for municipal CEFs, short-sale constraints deprive rational investors of arbitrage opportunities, thus allowing premiums to persist in CEF prices.[7] We provide a new perspective, indicating that short-sale constraints cause both premiums and discounts under hyperbolic discounting.

4 Conclusion

We show that hyperbolic discounting is a source of price discrepancies between CEFs and their UAs, known as the closed-end fund puzzle. Illiquid UAs, together with short-sale constraints on CEFs,

[5] Autore, Billingsley, and Kovacs (2011) report empirical evidence that positive abnormal stock returns were observed when the short-sale ban was implemented in the United States during the 2008 financial crisis.

[6] Based on a data set on short-sale costs in the New York Stock Exchange (NYSE) from 1926 to 1936, Jones and Lamont (2002) find that stocks with high short-sale costs are overpriced. Using high-frequency daily data on short-selling flows, Boehmer and Wu (2013) demonstrate that short sellers enhance the informational efficiency of domestic stock prices on the NYSE.

[7] See Kim and Lee (2007) as well, who present a model with limited market participation as a source of the CEF price discount and premium.

serve as a device for an individual's current self to make a commitment, thereby preventing the time-inconsistent behavior of future selves. Consequently, UA can be priced over CEF. We establish the conditions under which the CEF price discount arises even with no management fees on CEF. Moreover, we show that the current binding short-sale constraint on CEF generates the CEF price premium within a unified framework. Our findings offer new insights into the understanding of CEF pricing: the differences in the liquidity between the UAs and CEF is key for resolving the CEF puzzle.

Although our model explains the persistent discount (and premium) in CEF prices after an IPO, empirical studies demonstrate more complex price dynamics. Specifically, CEFs trade at a premium during the IPO, and subsequently, at a discount after 120 days on average (Rosenfeldt and Tuttle (1973), Weiss (1989), Lee, Shleifer, and Thaler (1991), Cherkes, Sagi, and Stanton (2009)). A comprehensive description of these phenomena requires considering not only hyperbolic discounting and commitment, but also other factors, such as underwriting and startup costs incurred at CEF IPOs (Lee, Shleifer, and Thaler (1991), Cherkes, Sagi, and Stanton (2009)). Incorporating risks into endowments and payoffs enriches the quantitative and qualitative implications. In addition, stochasticity renders commitment contingent on the realized state (Ogawa and Ohno (2024)), and generates fluctuations between the premium and discount on CEF prices.

Appendices

Appendix A: The relative magnitude of the threshold values

This appendix shows the relative magnitudes of the threshold values presented in Lemmas 2–5 and Proposition 1. Assumption 1 ensures the relationship (23) because

$$\begin{aligned}
&\frac{1+\beta_2\delta}{\left(\beta_2\delta+\beta_2\delta^2\right)\bar{r}}\Omega-\frac{1}{\delta\bar{r}}\Omega=\frac{1-\beta_2}{\left(\beta_2\delta+\beta_2\delta^2\right)\bar{r}}\Omega>0,\\
&\frac{1}{\delta\bar{r}}\Omega-\frac{\bar{r}-1}{\bar{r}}\Omega=\frac{1-\delta\left(\bar{r}-1\right)}{\delta\bar{r}}\Omega>0,\\
&\frac{\bar{r}-1}{\bar{r}}\Omega-\frac{\left(\beta_2\delta\bar{r}-1\right)\left(\bar{r}-1\right)}{1+\delta}\Omega=\frac{\left\{\left(1-\beta_2\right)\delta+\left(1+\bar{r}\right)\left[1-\beta_2\delta\left(\bar{r}-1\right)\right]\right\}\left(\bar{r}-1\right)}{\left(1+\delta\right)\bar{r}}\Omega>0,
\end{aligned}$$

where the signs of the second and third equations are determined using Assumption 1. Lemma 3 is based on the above last relationship.

When $\beta_2\bar{r}^2>1$, $\frac{(1+\bar{r})(\bar{r}-1)}{(\beta_2\bar{r}^2-1)\delta\bar{r}}\Omega$ is positive and satisfies the relationship (24) because

$$\frac{\left(1+\bar{r}\right)\left(\bar{r}-1\right)}{\left(\beta_2\bar{r}^2-1\right)\delta\bar{r}}\Omega-\frac{1+\beta_2\delta}{\left(\beta_2\delta+\beta_2\delta^2\right)\bar{r}}\Omega=\frac{\left(1-\beta_2\right)\left[\left(1+\beta_2\delta\right)+\beta_2\delta\left(1+\bar{r}\right)\left(\bar{r}-1\right)\right]}{\left(\beta_2\bar{r}^2-1\right)\left(\beta_2\delta+\beta_2\delta^2\right)\bar{r}}\Omega>0.$$

Combining (23) and (24), Lemma 2 uses the following relationship: When $\beta_2\bar{r}^2>1$,

$$\frac{\bar{r}-1}{\bar{r}}\Omega<\frac{\left(1+\bar{r}\right)\left(\bar{r}-1\right)}{\left(\beta_2\bar{r}^2-1\right)\delta\bar{r}}\Omega.$$

References

Amador Manuel, Ivàn Werning and George-Marios Angeletos, 2006, Commitment vs. Flexibility, *Econometrica,* 74, 365–396.

Attila Ambrus and Georgy Egorov, 2013, Comment on 'Commitment vs. Flexibility.' *Econometrica,* 81, 2113–2124.

Angeletos George-Marios, David Laibson, Andrea Repetto, Jeremy Tobacman and Stephen Weinberg, 2001. The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation, *Journal of Economic Perspectives,* 15, 47–68.

Autore M. Don, Billingsley, S. Randall, and Kovacs Tunde, 2011, The 2008 Short Sale Ban: Liquidity, Dispersion of Opinion, and the Cross-Section of Returns of US Financial Stocks, *Journal of Banking & Finance,* 35, 2252–2266.

Berk Jonathan B., and Richard Stanton, 2007, Managerial Ability, Compensation, and the Closed-End Fund Discount, *Journal of Finance,* 62, 529–556.

Boehmer Ekkehart and Juan Wu, 2013, Short Selling and the Price Discovery Process, *Review of Financial Studies,* 26, 287–322.

Brauer Greggory A., 1984, "Open-Ending" Closed-End Funds. *Journal of Financial Economics,* 13, 491–507.

Brauer Greggory A., 1988, Closed-End Fund Shares' Abnormal Returns and the Perceived Quality of Investment Advice, *Journal of Finance,* 43, 113–127.

Brickley James A. and James S. Schallheim, 1985, Lifting the Lid on Closed-End Investment Companies: A Case of Abnormal Returns. *Journal of Financial and Quantitative Analysis,* 20, 103–117.

Bryan Gharad, Dean Karlan and Scott Nelson, 2010, Commitment Devices. *Annual Review of Economics,* 2, 671–698.

Cherkes Martin, 2012, Closed-End Funds: A Survey, *Annual Review of Financial Economics,* 4, 431–445.

Cherkes Martin, Jacob Sagi and Richard Stanton, 2009, A Liquidity-Based Theory of Closed-End Funds, *Review of Financial Studies,* 22, 257–297.

De Long Bradford J., Andrei Shleifer, Lawrence Summers and Robert Waldmann, (1990). Noise Trader Risk in Financial Markets, *Journal of Political Economy,* 98, 703–738.

Diamond Douglas and Robert E. Verrecchia, 1987, Constraints on Short-Selling and Asset Price Adjustment to Private Information, *Journal of Financial Economics,* 18, 277–311.

Ghent Andra, 2015, Home Ownership, Household Leverage and Hyperbolic Discounting. *Real Estate Economics,* 43, 750–781.

Herzfeld J. Thomas, 1980, The Investor's Guide to Closed-End Funds. (McGraw-Hill, New York, NY).

Jones Charles and Owen Lamont, 2002, Short-Sale Constraints and Stock Returns. *Journal of Financial Economics,* 66, 207–239.

Kang Jingoo and Minwook Kang, 2022, Durable Goods as Commitment Devices under Quasi-Hyperbolic Discounting. *Journal of Mathematical Economics,* 99, 102561.

Kim Youngsoo and Bong Soo Lee, 2007, Limited Participation and the Closed-End Fund Discount. *Journal of Banking & Finance,* 31, 381–399.

Laibson David, 1997, Golden Eggs and Hyperbolic Discounting. *Quarterly Journal of Economics,* 112, 443–477.

Charles Lee, Shleifer Andrei and Thaler H. Richard, 1991, Investor Sentiment and the Closed-End Fund Puzzle, *Journal of Finance,* 46, 75–109.

Malin Benjamin, 2008, Hyperbolic Discounting and Uniform Savings Floors, *Journal of Public Economics,* 92, 1986–2002.

Miller Edward M., 1977, Risk, Uncertainty, and Divergence of Opinion. *The Journal of Finance,* 32, 1151–1168.

Nocke Volker and Martin Peitz, 2003, Hyperbolic Discounting and Secondary Markets. *Games and Economic Behavior,* 44, 77–97.

O'Donoghue Ted and Matthew Rabin, 1999, Doing It Now or Later. *American Economic Review,* 89, 103–124.

O'Donoghue Ted and Matthew Rabin, 2001, Choice and Procrastination. *Quarterly Journal of Economics,* 116, 121–160.

Ogawa Takayuki and Hiroaki Ohno, 2024, Hyperbolic Discounting and State-Dependent Commitment. *Economica,* 91, 414–445.

Pollak Robert, 1968, Consistent Planning. *Review of Economic Studies,* 35, 201–208.

Pontiff Jeffrey, 1996, Costly Arbitrage: Evidence from Closed-End Funds. *Quarterly Journal of Economics,* 111, 1135–1151.

Phelps Edmund and Robert Pollak, 1968, On Second-Best National Saving and Game-Equilibrium Growth. *Review of Economic Studies,* 35, 185–199.

Richards R. Malcolm, Donald R. Fraser and John C Groth, 1980, Winning Strategies for Closed-End Funds. *Journal of Portfolio Management,* 7, 50–55.

Roenfeldt Rodney L. and Donald L. Tuttle, 1973, An Examination of the Discounts and Premiums of Closed-End Investment Companies. *Journal of Business Research,* 1, 129–140.

Schlafmann Kathrin, 2021, Housing, Mortgages, and Self-Control. *Review of Financial Studies,* 34, 2648–2687.

Schwarz Mordechai E. and Eytan Sheshinski, 2007, Quasi-Hyperbolic Discounting and Social Security Systems. *European Economic Review,* 51, 1247–1262.

Strotz Robert H., 1956, Myopia and Inconsistency in Dynamic Utility Maximization. *Review of Economic Studies,* 23, 165–180.

Thompson Rex, 1978, The Information Content of Discounts and Premiums on Closed-End Fund Shares. *Journal of Financial Economics,* 6, 151–186.

Tirole, Jean, 1985, Asset Bubbles and Overlapping Generations. *Econometrica,* 53, 1499–1528.

Weiss Kathleen, 1989, The Post-Offering Price Performance of Closed-End Funds. *Financial Management,* 18, 57–67.