Re-examining Edgeworth’s “Field of Competition”: Towards Reconstructing the Vision of Competitive Markets

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This paper offers a unified reinterpretation of Edgeworth (1881) and Edgeworth (1925 [1897]) and presents a perspective that views competition not as a search for an equilibrium point, but as a competitive process in motion through continual re-contracting processes within Edgeworth’s ‘filed of competition’.

1. Introduction

1-1 The Problem: Is Competition Equilibrium or Movement?

The question of how to understand competition is one of the most fundamental issues in economics. Within the mainstream modern framework, competition is typically understood through the concept of equilibrium. That is to say, competition is understood as a problem that presupposes a consistent state (equilibrium) to be ultimately reached, and seeks to clarify its existence and nature. In other words, the point at which the optimal responses of each agent converge, or the equilibrium path as its dynamic extension, is positioned as the outcome of competition. Under this view, price adjustments and changes in strategy are understood as the process leading to the final equilibrium, or as a transition between equilibria. In particular, in perfect competition, all economic agents act as price-takers, and if competition is imperfect, that state is viewed as a deviation from the competitive equilibrium point.

However, when viewed in the light of F.Y. Edgeworth’s analysis, this conception of competition requires a fundamental re-examination. *Mathematical Psychics* (1881) was later formalised as

'Edgeworth's Limit Theorem' and re-evaluated as the foundation of competitive equilibrium. However, this re-evaluation emphasized only one aspect of Edgeworth's analysis in *Mathematical Psychics* (1881), and it is difficult to say that it captures the full picture of his understanding of competition.

The contract theory in *Mathematical Psychics* (1881) referred to his contemporary physics and its analysis (the calculus of variations), addressing both the problem of arbitrage based on utilitarianism and the problem of economic calculation based on Jevons's theory of exchange. As is well known in the history of economics, Edgeworth engaged in a debate with A. Marshall on the barter problem whether the transaction process is path dependent or not. He also debated L. Walras, via L. von Woltkevitz, regarding profits in the competitive process, which Edgeworth regard as not vanishing in the competitive process. In fact, these controversies suggested that Edgeworth's analysis contained fundamental differences from the equilibrium theory that was taking shape during the Marginal Revolution in terms of the understanding of the competitive process (see Nakano (2015)). Although the dispute between Edgeworth and Marshall in one hand, Walras in another, was quite serious, combined with Edgeworth's disposition to avoid emotional confrontation, they never resolved their theoretical differences. These differences have been overlooked to this day without being fully understood.

Nevertheless, in recent years, from the 1980s to the 2020s, attention has been drawn to the fact that the phenomenon known as the Edgeworth cycle and Edgeworth's taxation paradox is present in Edgeworth's paper written in Italian (Edgeworth (1897)), and attempts have been made to formulate it in modern terms. This Italian paper was reprinted in English in 1925 in a collection of economic papers edited by Edgeworth himself (Edgeworth (1925)[1897]). This paper interprets Edgeworth ((1925)[1897]) as clarifying the differences between his own approach and the equilibrium theory, and by extension articulating his views on Bertrand and Cournot's discussions of oligopolistic markets. Furthermore, this paper attempts to read *Mathematical Psychics* (1881) and Edgeworth (1925 [1897]) in a unified manner, thereby seeking to elucidate Edgeworth's view of competition based on his concept of the 'field of competition'.

In Edgeworth's view, competition is not necessarily depicted as a process of convergence towards an equilibrium point. Rather, exchange and price adjustment appear as a movement that continues to change persistently under certain conditions. The starting point of this paper is to re-examine this difference head-on. In particular, it reinterprets Edgeworth's cycle not as a 'special form of equilibrium' but as a 'process in motion emerging in the absence of equilibrium'. The aim is to clearly position the underlying understanding of competition as a framework distinct from equilibrium theory.

1-2 The Distinctiveness of Edgeworth: Process Rather Than a Point

A distinctive feature of Edgeworth's analysis lies in his treatment of exchange and prices not as a single point solution, but as a continuous process of change. In his 1881 analysis, he introduced the contract curve, illustrating the set of points where utility (indifference curves) is tangent. Crucially, however, this curve is not presented merely as a 'set of final points of agreement'. As long as the possibility of renegotiation remains, exchange continues to evolve. The contract curve is not an endpoint but rather functions as a pathway along which exchange proceeds. This perspective becomes even clearer in his 1925 analysis of price competition. There, price adjustments between firms do not converge on a single equilibrium point but rather manifest as continuous fluctuations, so-called 'cycles'. This cycle should be understood not as a form of equilibrium that is stably maintained, but rather as a sustained movement arising in a situation where equilibrium does not hold. Consequently, for Edgeworth, the basic unit for understanding competition is not the choice of the agent, but the direction of change determined by each state. In this context, competition is grasped not as a set of equilibrium points, but as a field of motion within the state space.

1-3 Intellectual Background: The Field Perspective in Physics

Although this understanding of competition was unusual within the economics of the time, it resonated with the view that was developing in his contemporary physics as the field theory of classical mechanics[1]. In the late 19th century, physics—particularly the theories represented by J.C. Maxwell and W. Thomson—was evolving towards a view that treated phenomena not as solutions to individual equations, but as fields defined across the entire space[2]. In this context, the problem lay not in determining a specific point, but in describing what forces or directions of change are determined at each point in space. Electric and magnetic fields possess a structure as a whole, rather than being states at a single point, and it is within this structure that the motion of particles is determined. In this context, motion is not directed towards a goal imposed from outside, but rather constitutes a

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1 Of the approximately 50 references cited in Edgeworth (1881), one fourth of them (12 papers and books, which are listed in the end of this paper) relate to physics and analytical mathematics, in particular, the calculus of variations. In particular, he refers to the works of Todhunter (1871), Tait and Thomson (1879), and Maxwell and Thomson. There are around ten mentions to each of these works. In other words, Edgeworth was attempting to apply their concepts and technics to economics whilst closely following the detail, where classical mechanics and electromagnetism were becoming increasingly linked to the calculus of variations.

2 In physics, Newtonian mechanics initially developed as a theory for analysing 'what forces act on individual particles'. Within this framework, gravity was understood as a force acting directly between objects, and motion was described by the equation of motion $F = ma$. The focus of the analysis was on the forces acting on a particle at any given moment and the resulting acceleration. However, in the 19th century, with the development of electromagnetism, the question of 'how forces propagate through space' came to the fore. Faraday and Maxwell conceived of gravity and electromagnetic forces not merely as direct interactions between objects, but as 'fields' distributed throughout space itself. Here, the direction and magnitude of the force are defined at each point, and particles move in accordance with that field. Consequently, the focus of analysis shifts from individual particles to the 'structure of the field extending throughout space'. Furthermore, in the calculus of variations developed by Lagrange and Hamilton, motion came to be understood not in terms of the forces at each instant, but as a trajectory that minimises a quantity known as the 'action'. In other words, nature is thought to choose a path that minimises (or stabilises) a certain quantity within the entire motion, including the starting and ending points, rather than simply obeying forces locally. The concept became the foundation of modern physics. In the general theory of relativity, and furthermore, in quantum mechanics and quantum field theory, the field becomes the fundamental object rather than the particle itself. Consequently, the development of physics can be understood, broadly speaking, as a process in which the object of analysis was extended from 'local forces acting on point particles' to 'fields extending throughout space', and further to 'principles of action that determine the entire motion'. I Referred to Yamamoto (1997) on the history of Mechanics.

trajectory that arises naturally in accordance with the structure of the field. Physics was developing field theory by integrating with analysis, centred on the calculus of variations.

Edgeworth's analysis resonates deeply with this perspective. He viewed exchange and price adjustments not as the determination of a final equilibrium point, but as a process in which the direction of movement varies from one state to the next. Whilst the contract curve is formulated as a problem of 'contact' conditions regarding the utility or profit of economic agents, that contract curve continues to evolve through the process of renegotiation. Edgeworth's limit theorem viewed the convergence process as the movement of the contract curve contracting towards the competitive equilibrium point through renegotiation. However, it is possible that not only the movement of the convergence process towards competitive equilibrium, but also a cyclical rotation process may emerge. This is the discussion of cycles found in Edgeworth (1925) [1897]. What is important here is that the relationship with physics is not merely a metaphor. For Edgeworth, the basic unit for understanding competition is not the choice of the agent, but the direction of change determined by the state. In this sense, his theory belongs to a cognitive framework distinct from that of equilibrium-centred economics. Edgeworth drew on the methods of physics, whilst making it clear that economics is not necessarily isomorphic with physics, and sought to transplant them into economics.[3]

1-4 The Argument of this Paper: A Reinterpretation as a Competitive Field

In light of the above, this paper puts forward the following arguments.

First, Edgeworth's analyses of (1881) and Edgeworth(1925 [1897])are not independent of one another but should be understood as a unified whole within the common framework of 'conceiving competition as the movement of a field'. The contract curve of 1881 provides a pathway within which movement is constrained, whilst the cycle of 1925 illustrates rotational movement within that field.

Second, Edgeworth's cycle is not a form of equilibrium, but rather a consequence of the fact that the space describing the competitive process does not possess a simple optimisation structure. That is to say, competition does not necessarily converge to equilibrium, but may instead manifest as sustained fluctuation.

Thirdly, this understanding differs fundamentally from modern game theory, particularly the dynamic price competition model proposed by Maskin and Tirole (1988). Whereas in Maskin and

[3] Mirowsky, P. (1989) conducted a comprehensive study of the relationship between physics and economics, confirming the strong influence of physics on economics. However, Edgeworth did not simply imitate the methods of physics; rather, with structural analogies in mind, he constructed a unique logic as a theory of contracts in the market. The tradition of advancing scientific research through the method of analogy can be seen in John F. W. Herschel, J. C. Maxwell and W. S. Jevons who was influenced by them. I intend to explore the possibility of developing this paper in relation to the history of science in a separate paper.

Tirole (1988) the cycle is constructed as a consequence of equilibrium strategies, in Edgeworth the cycle is a movement that arises inevitably from the very structure of the competitive process itself.

Fourthly, Armstrong and Vickers (2023) provide a modern analysis of the tax paradox noted by Edgeworth (1925 [1897]), focusing on the complex behaviour of the effects brought about by increases in taxes or costs when there are producers supplying across multiple goods. This is particularly important when considering competition policy in markets where modern platform operators exist. We note that analysing competitive markets from this perspective may lead to a revival of Edgeworth's view of the competitive field.

This paper argues that Edgeworth's theory is not an incomplete form of equilibrium theory, but rather an independent theoretical framework that conceives of competition as a process in motion, and that it offers a perspective of great importance for understanding modern competitive markets.

1-5 Structure of this paper

The structure of this paper is as follows. Section 2 reinterprets the 1881 theory of contracts, positioning the contract curve as a pathway of motion. Section 3 analyses the 1925 theory of price competition, viewing the cycle as a rotational movement within the competitive field. Section 4 integrates these two approaches to interpret Edgeworth's concept of the 'competitive field'. Section 5 clarifies the differences from equilibrium theory through a comparison with Maskin–Tirole (1988). Section 6 demonstrates the validity of this framework using examples such as the tax paradox and platform firms. Consequently, this paper attempts to redefine competition not as an equilibrium problem, but as a movement within the field.

2. The Concept of Contractual Movement in Edgeworth (1881)

Edgeworth's contract theory in *Mathematical Psychics* (1881) has often been interpreted as a static equilibrium theory centred on the contract curve, or as a kind of cooperative game[4]. However, a close examination of the text reveals that Edgeworth conceived of the contract not merely as the determination of an equilibrium point, but as a movement within the competitive field that is directed, constrained, and moving towards a state of rest. This interpretation does not imply that he provided explicit dynamical equations. Rather, when we synthesise his vocabulary concerning contracts, his geometric derivations, his definition of the competitive field, the relationship between the demand curve and the contract curve, and the variational analogy in Appendix V, it becomes clear that contracts are understood not as 'points' but as 'movement within a mobile domain'.

2-1 M ècanique Sociale as an Analogy to the Principle of Least Action

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[4] For example, see Shubik, M. (1959)

To understand this point, we must first confirm the methodological starting point of *Mathematical Physics* as a whole. At the beginning of the theoretical section, Edgeworth explains the possibility of mathematical reasoning in the social sciences by drawing an analogy with the principle of maximum energy (principle of least action) in physics. According to him, the main problem in the social sciences, he argues, is that, just as in physical phenomena, society is equipped with mechanisms that generate the maximum energy within its interactions. Economics deals with the arrangement whereby each agent seeks to maximise their own utility, whilst political science and utilitarian ethics deal with the arrangement whereby the total utility of the whole is maximised. He states that this structure is similar to the calculus of variations, writing: "Social Science, as compared with the Calculus of Variations, starts from similar data … and travels to a similar conclusion—determination of maximum." (Edgeworth (1881) p.7) What is important here is that Edgeworth positions economics and ethics not as algebraic solutions within a system of quantitative equations, but as the calculus of variations based on analysis grounded in the principle of least action.

This methodological stance is further clarified by the references to Lagrange and Hamilton on pp. 10–11. Edgeworth describes the structure of mechanics in which, in the interaction of complex bodies, the motion of each particle follows the maximisation of 'Action—energy integrated over time', and explains that Hamilton made it possible to 'derive the velocities of the parts from the total action'. He goes on to state that this time-integrated energy is analogous to the total amount of pleasure accumulated over time. In other words, the concept of action in physics is correlated with the concept of cumulative pleasure in utilitarianism[5]. Edgeworth also states that, in economics, the arrangement towards which self-interested contractors move can be understood as a 'relative maximum' under certain conditions.

At this stage, contract theory is no longer a static point theory but is situated against a background of Mechanical analogy. However, Edgeworth is not simply applying the principle of least action directly to economic phenomena. Rather, what he is doing is adopting the method of physics—which treats complex interactions as structures involving forces and actions that cannot be directly measured—as a model for the mathematisation of the social sciences. In this sense, the references on pp. 14–15 to vortex motion, minimum potential and minimum momentum-potential, whilst not providing exact equations for contractual motion, do form a conceptual foundation for understanding the interactions of economic agents as 'directional motion'. Edgeworth conceives of pleasure as a 'pleasure-force', explaining that it controls a more coarse-grained energy and directs the agent's

[5] The first economic paper to apply the calculus of variations in the history of economics was that of Ramsay (1928). It pioneered the theory of optimal growth. It is likely that Ramsay was influenced to some extent by Edgeworth, perhaps via Pigou. However, as Ramsay focused strictly on the problem of the macroeconomic optimal path, Edgeworth's perspective is not reflected in his work. Although Hotelling (1929) was under the influence of Edgeworth to stabilize the instability identified by Edgeworth, Hotelling developed his original theory of location competition. Hotelling (1932) expanded upon the discussion of the taxation paradox put forward by Edgeworth (1925 [1897]). Wade Hands (1998) and Wade Hands, D and P. Mirowsky (1998) indicated that Hotelling's theory of market price , which reflected the potential theory in physics has been neglected in the history of economics.

behaviour. Furthermore, he depicts the subject matter of the social sciences as a system of 'charioteers and chariots' (Edgeworth (1881) p.15)—that is, a system of forces that attract, collide and combine with one another—and states that this exhibits quantitative regularities similar to those of an electromagnetic field.

2-2 Contracts and the field of Competition

Building on this methodological premise, in the economic calculation of Part II, the contract is first defined in opposition to 'war'. According to Edgeworth, an act based on self-interest is 'war' when carried out without the consent of others, and a 'contract' when it involves the consent of others. (Edgeworth (1881) p.17) Furthermore, competition is conceived as a process in which peace, brought about by agreed contracts, intersects with war, which involves renegotiating contracts without the consent of others. Here, a contract is defined not as a static state established once and for all, but as a process that can be altered through renegotiation. He contrasts 'contractors during contract' with 'recontract', viewing competition as a process capable of modifying existing contracts.

The concept that generalises this possibility of re-contracting is the 'field of competition'. Edgeworth defines the field of competition regarding a given contract as 'all the individuals who are willing and able to recontract about the articles under consideration" (Edgeworth (1881) p.17). That is, the field consists of traders possessing the will and ability to re-contract regarding the subject matter of contracts. Consequently, the field of competition is not merely a collection of market participants, but the scope of potential interactions capable of altering existing contracts. Furthermore, he explicitly draws an analogy between this field and the 'field of force' in physics.

> "Thus, if one chose to define the field of force as the centers of force sensibly acting on a certain system bodies, then in a continuous medium of attracting matter, the field might be continually of indefinite extent, might change as the system moved, might be said to vanish when the system reached equilibrium, " (Edgeworth (1881) p.18)

That is to say, if one considers a field in which gravitational, magnetic, or electromagnetic forces act upon a system of objects, he states that 'the field changes as the system moves, and can be said to disappear once the system reaches equilibrium'. Here, Edgeworth conceives of the field of competition not as a fixed set, but as a structure that changes in tandem with the motion of the system.

This point is decisive for understanding the concept of the movement of contracts. For if the competitive field is a 'set of subjects capable of renegotiation' and, moreover, changes in response to the system's movement, then contracts are understood not as occurring at a single point within a fixed space, but as proceeding within a variable field. The competitive field is not a field in the strict sense of physics, where force vectors are assigned to each point[6]. However, the competitive field possesses a structure that determines which re-contracts are possible and in which directions changes to contracts are permitted. In this sense, the competitive field is a quasi-dynamic field that offers the possibility of conceiving contracts as being in a state of motion.

2-3 Direction, Force and Motion of Contracts

Next, the aspect of movement in motion becomes even clearer when we examine how Edgeworth derives the contract curve. He considers a bilateral contract, setting the utility of one party as $P = F(x, y)$ and that of the other as $\Pi = \Phi(x, y)$ (Edgeworth (1881) p.20). He then formulates the problem of finding the contract curve not simply as the problem of finding the point that maximises utility, but as the problem of an infinitesimal shift from any given point. He states "It is required to find a point (x y) such that, in whatever direction we take an infinitely small step, P and Π do not increase together, but that, while one increases, the other decreases. " (Edgeworth (1881) p.21). That is, the point to be sought is the one where no matter in which direction an infinitesimal step is taken, P and Π do not increase simultaneously. In other words, the contract curve is defined from the outset by the presence or absence of 'directions in which movement is possible'.

What is significant in Edgeworth's contract theory is that the contract curve is not presented merely as an equilibrium set, but is derived as a boundary delimiting the range within which the contract can move. When considering a bilateral contract, he does not first ask 'which point is optimal', but rather asks in which directions an infinitesimal movement is possible from any given point. In other words, the question is whether, when taking an infinitesimal step from a given point, there exists a direction in which the utility of both parties can increase simultaneously. From this question, the contract curve is derived as the trajectory of points where directions in which both parties can improve simultaneously cease to exist. Therefore, the contract curve is, in modern terms, a set of Pareto-efficient points; however, in accordance with Edgeworth's description, it is first and foremost a set of stationary points where directions of movement disappear. [7]

This point is further clarified by his terminology of the 'line of force' or 'line of preference'. Each agent is depicted not as stationary along an indifference curve, but as possessing a direction of preference that is orthogonal to the indifference curve. The two agents can both modify their contracts only when a positive direction remains between their respective preference directions. Conversely, when their indifference curves touch and their preference directions are opposite, a mutually beneficial

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[6] Pikler, Andrew G. (1954) and Pikler, Andrew G. (1955) interpret Edgeworth's theory of utility as a discussion of a 'field of utility' analogous to physical phenomena. However, Edgeworth does not develop the 'field of utility' as a direct analogy to physical phenomena such as magnetic fields. Rather, the analogy of a 'field' is employed within the specific logic of the economic phenomenon known as the 'field of competition'.

[7] Morgan, M. S. (2012) suggested the difference of the meaning of Edgeworth Box in textbooks and Edgeworth original diagram. Modern microeconomics textbooks follow mostly the tradition of Pareto. Nakano (2024) searched, "Why didn't Edgeworth draw the Edgeworth Box?" , which presented in 2024 ESHET annual conference held at Graz.

infinitesimal shift becomes impossible. In this sense, the contract curve does not impose an endpoint for exchange from the outside, but arises endogenously from the directional structure within the movable domain.

In the case of a bilateral contract, this movable domain is represented as a so-called football-shaped region, bounded by the indifference curves of the two parties passing through the initial allocation. This region is not merely an illustrative diagram. It represents the range of contracts that can be moved by mutual consent, starting from the initial contract. Within this region, as there may be multiple directions that improve the situation for both parties, the path that the contract will follow is not uniquely determined. Therefore, Edgeworth's contract theory is not a theory that determines a single point on the contract curve, but rather a theory that defines the region within which the contract can move and the boundary where that movement may cease.

2-4 The Field of Competition and the Transformation of Contractual Movement

When competitions introduced, this area of movement undergoes further transformation. What Edgeworth calls the 'field of competition' is the range of agents who possess the will and ability to renegotiate a given contract; it is not a fixed set of market participants. He draws an analogy between this field and the 'field of force' in physics, stating that the field may change as the system moves. Consequently, the field of competition is not the external environment surrounding the contract movement, but rather a structure that determines which renegotiations are possible and in which directions movement is permitted.

This structure is embodied in the system of supplementary contract curves. These are not merely auxiliary diagrams attached to the main contract curve, but a mechanism that demonstrates which renegotiations become possible and which final contracts are excluded as the number of competitors or their combinations change. For example, when a second X and a second Y are added to a two-party contract, points that were previously close to the end of the contract curve might have been final settlements; however, as new possibilities for renegotiation arise, these points cease to be stable final contracts. Edgeworth describes this process as the system being 'worked down' to the contract curve (Edgeworth (1881) p.37, 38, 41). Here, 'worked down' does not simply mean that the final result shifts downwards; rather, it refers to the process whereby the range of possibilities is eroded by competitive pressure, and the indeterminate interval remaining on the contract curve is narrowed.

Consequently, the role of supplementary contract curves is not to directly determine the final point on the contract curve, but to reorganize the movable region and the residual indeterminate region in response to changes in the competitive field. If the number of competitors increases, the likelihood of renegotiation rises, and the permissible range on the contract curve narrows. Conversely, if competition is imperfect, not all final settlements are eliminated, and a finite indeterminate interval remains on the contract curve. It is precisely this residual interval that Edgeworth identified as a characteristic of imperfect competition.

2-5 Imperfect Competition and the Interpretation of the Demand Curve

In light of this, Edgeworth's demand curve also has a different meaning from that of a standard market demand function. In perfect competition, individual demand curves are aggregated to form a standard market demand curve that determines the equilibrium between supply and demand. However, Edgeworth's demand curve is, more fundamentally, the trajectory of the points at which the utility of the principal agent remains constant at each exchange ratio. In other words, it is a local directional condition indicating how far the agent can move along a given direction—that of the exchange ratio. In imperfect competition, since price ratios are not uniquely fixed, the demand curve does not serve as a mechanism for directly determining the equilibrium point, but rather functions as a geometric condition constraining movement within a movable domain.

From the above, Edgeworth's 1881 argument is not a theory that determines the entire path of the contract movement, but rather a theory that clarifies the domain within which contracts can move, the competitive field that shapes that domain, and the residual indeterminate interval on the contract curve where the movement may cease. As long as this residual indeterminate interval remains, the movement of contracts is not absorbed into a single equilibrium point. Rather, even after the range of movement has been reduced by competition, multiple final settlements remain possible. The Edgeworth cycle, which we shall examine in Section 3, can be interpreted as the problem that prices or contracts can move cyclically within the indeterminate region left by this 1881 theory. In other words, the 1925 cycle theory does not represent a break with the 1881 theory of contract movement; rather, it can be understood as an elaboration, in the form of a cyclical movement over time, of the mobility left on the contract curve in 1881—that is, the residual indeterminacy that is not eliminated even by competition.

2-6 The Cycloid Analogy from Jevons' Exchange Equation

The discussion in Edgeworth's Appendix V, 'On Professor Jevons's Formulae of Exchange', provides a corresponding mathematical framework for the structure of the contract movement outlined above. To understand this point, it is necessary to view the reference to the calculus of variations as an analogy to the principle of least action, and to regard the competitive field and the directional contract movement as continuous.

As confirmed in Section 2-1, Edgeworth analogises problems in the social sciences to the maximisation principle, namely the principle of least action. However, the maximisation envisaged here is not optimisation at a single point. As seen in Section 1-3, a contract is a movement that can have multiple directions of improvement within a domain of motion, and its path is not uniquely determined. Furthermore, as shown in 2-2, this movement is constrained by the competitive field, and the domain of motion itself undergoes deformation. Consequently, the problem lies not **in** finding a specific point, but in describing the conditions that hold within the set of permissible movements.

The mathematical method corresponding to this problem structure is the calculus of variations. A variational problem is not one of maximising a single value, but of finding, from a set of permissible functions or paths, those that satisfy the conditions. Edgeworth cites the cycloid as an example precisely to illustrate this problem structure (Edgeworth (1881) p.111). Whilst the cycloid is known as a solution to the brachistochrone problem, what is important here is not its specific shape, but the fact that, amongst multiple paths, a path is characterised by specific conditions.

Edgeworth applies this structure to Jevons' exchange equations[8]. In Jevons' framework, the equilibrium of exchange is given as an algebraic condition: the equality of marginal rates of utility. However, Edgeworth reinterprets this as a condition of contact between the families of indifference curves of each agent. That is to say, equilibrium is the point at which indifference curves share the same tangent line at a single point; this is not merely a numerical condition, but implies a convergence of direction. At this point, the problem ceases to be one of solving for a specific point, and instead deals with the geometric conditions defined by the contact between families of curves.

Through this reinterpretation, the contract is understood not merely as an equilibrium point, but as a combination of motion within a movable domain, together with directional conditions and stopping conditions for that motion. That is to say, the contract motion may have multiple paths, but that motion is constrained at each point by the direction of the tangent to the indifference curve, and ultimately comes to rest on the contract curve where the contact condition holds. In this sense, the cycloid analogy does not imply that exchange follows a specific curve, but rather indicates that exchange should be formulated as a path problem.

Therefore, Appendix V reinterprets the contract as a problem of motion rather than a problem of points by situating Jevons's exchange equation within a variational framework. This reinterpretation is consistent with the deformation of the movable domain due to the competitive field, as shown in 2-2, and with the contract motion following a direction, as shown in 2-3; it provides the mathematical foundation for a unified understanding of Edgeworth's entire theory as a theory of constrained motion within the movable domain.

To summarise the above arguments, Edgeworth's contract theory should be understood not merely as a theory of equilibrium points, but as a theory of motion within the movable region. Firstly, in the theoretical section, he conceives of the social sciences in terms of analogies with the principles of maximisation and minimisation, action, energy, and the pleasure principle.Secondly, in economic calculation, he defines a contract as a process that can be renegotiated, and conceives the competitive

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[8] As to the relationship between Walras equilibrium and Jevons exchange equilibrium, see Nakano (2009). Jevons exchange equilibrium does not always coincide with Walras equilibrium in the case of multiple traders and goods. Jevons's reference to the Principles of Virtual Velocity may have suggested to the way how Edgeworth had an idea of applying the Principle of Least Action to the trading processes.

field as a field that changes in response to the system's motion. Thirdly, the contract curve is derived as a set of points where directions of improvement for both parties cease to exist, and is explained using the vocabulary of direction, force and motion. Fourthly, changes in the number of competitors shrink or deform the movable domain, causing the system to be 'worked down' to the contract curve.Fifthly, in imperfect competition, contracts are path-dependent because they 'run down' to different final settlements depending on the initial position. Sixthly, in Appendix V, Jevons's exchange equation is reinterpreted as a contact condition for a family of curves, and further as a variational analogy.

Consequently, a contract in Edgeworth's framework is not a pre-determined equilibrium point. It is a movement within the competitive field, determined by the direction of each agent's preferences, the possibility of renegotiation, local constraints imposed by demand curves, and stopping conditions imposed by the contract curve. His theory does not constitute a set of dynamical equations in the strict sense. However, it provides a framework for analysing contracts not merely in terms of 'where equilibrium is reached', but also in terms of 'within what range, in what direction, and under what constraints they can move'. It is in this respect that the internal connections between the contract curve, the demand curve, the field of competition, and the variational analogy in *Mathematical Psychics* lie.

3. Edgeworth (1925 [1897]): The Cycle of Competition and the Market as a Process

Edgeworth republished a paper he had written in Italian in 1897, adding an English translation and making some alterations and corrections when he edited a collection of his economic papers in 1925, whilst retaining the main points. The overall thrust of the work appears to be a complement to Marshall's monopoly theory of taxation and subsidy policy based on partial equilibrium analysis. However, the main arguments put forward are: (1) in the case of multiple monopolies, equilibrium is indeterminate (which is fundamentally different from competition); and (2) the effects of policy are counterintuitive (taxes may sometimes increase consumer welfare). Furthermore, a closer examination of the analysis reveals that Edgeworth thoroughly applied the approach taken in his *1881 paper* in *Mathematical Psychics* (1881). Consequently, Edgeworth highlighted two points that are referenced in modern theoretical research:First, he identified the cyclical phenomenon now recognised as the Edgeworth cycle. Second, he presented the problem that has recently come to be known as the Edgeworth taxation paradox.

In this section, we organise the theory of multiple monopolies in Edgeworth (1925 [1897]) from three perspectives: (1) an accurate reconstruction of its content; (2) its relationship with *Mathematical Psychics* (1881); and (3) its relationship with the Cournot and Bertrand theory. On this basis, we will clarify how the central argument of this paper—the understanding of 'competition as movement within a field'—manifests itself in the 1925 discussion.

3-1 The Basic Problem Formulation in Edgeworth (1925): The Indeterminacy of Equilibrium

The central argument of Edgeworth (1925) is that, under multiple monopolies, prices do not converge to a single stable point. He demonstrated that, in a market consisting of two monopolists and a multitude of competing demanders, price competition does not have a single 'final settlement' but continues to fluctuate within a certain range.

In Edgeworth (1881), he developed an argument regarding how the state of contracts evolve through changes in the competitive field, centring on a contract curve where the gradients of marginal rates of substitution between agents coincide, based on the agents' utility evaluations. In Edgeworth (1925), he addressed a market in which two profit-maximising monopolistic producers (an oligopolistic market) transact with a multitude of consumers. In this context, position-dependent evaluations held by each agent—such as profit or utility—can be understood as local 'potentials' that determine the direction of the agent's behaviour. Here, 'potential' does not refer merely to the level of profit itself, but rather to the gradient of the marginal evaluation that indicates which direction of movement from the current state would be more advantageous. Normally, given the behaviour of other agents, each agent acts in accordance with this local gradient; as a result, the set of points where marginal conditions coincide appears as the contract curves or intersection of optimal reaction curves. As long as the structure of the given market is fixed, this point functions as the endpoint of spontaneous trade.

However, what Edgeworth (1925) demonstrated is that this 'terminality' does not generally hold. Particularly in oligopolistic markets, each agent's profit conditions depend on the behaviour of other agents, and a price change or contract modification by one agent alters the profit gradient for the other agent. Consequently, even if marginal conditions are temporarily satisfied at a given point, the shape of the potential itself changes through the other party's reaction, giving rise to a renewed incentive to move. As a result, rather than converging on a fixed equilibrium point, agents move within a potential field that continues to be reshaped by their interactions.

Edgeworth demonstrates that uncertainty arises not only with perfectly identical goods but also with rival goods. In other words, even if there is no perfect match, equilibrium will not be stable as long as demand is competitive. Furthermore, the situation is somewhat different in the case of complementary goods. These are goods that are interdependent, such as a factory and its power source. In this case, the Cournotian notion that 'equilibrium is determined' may at first glance appear plausible , but Edgeworth demonstrated that, in reality, strategic interaction continues because the monopolist sets prices in consideration of the other party's future behaviour, meaning that equilibrium may not be fixed.

3-2 The Cycle in the Case of Substitute Goods: The Dynamics of Price Competition

In the case of two monopolists supplying identical goods, i.e. perfectly substitutable goods (pp. 118–121), Edgeworth describes a specific price movement.

Initially, each monopolist sets a monopoly price for its own customers. However, one monopolist has an incentive to attract the rival's customers by offering a price slightly lower than the other's. Consequently, a price-cutting competition begins. This process continues until prices reach a low level (point Q in Figure 2). However, even in this state, equilibrium is not achieved. As each monopolist faces supply constraints, neither can satisfy the total market demand. Consequently, an incentive arises once again to sell at a higher price to meet the remaining unmet demand. Prices then rise, and a new round of price-cutting competition begins.

This movement is depicted not merely as an adjustment process, but as a movement that theoretically has no stopping point. Edgeworth describes this state as follows.

> "In the latter case, there will be an indeterminate range within which the index of value will oscillate, or rather will fluctuate irregularly for an indefinite period of time." "That definite state of equilibrium characteristic of perfect competition will never be attained..." (Edgeworth 1925, p. 118)

It is worth noting here that Edgeworth does not use the term 'cycle', as is common in modern discourse. Instead, he employs the terms 'oscillate' and 'vibrate', describing the movement as oscillating within a state space with the nuance, as used in physics, of 'moving from one position to another and back again'.

In the case of substitute goods, competition manifests itself primarily as a struggle for demand. Since a price reduction by one party directly reduces demand for the other, there is a continuous downward pressure on prices. However, where supply constraints exist, demand cannot be entirely captured, leaving the possibility of renegotiation over unmet demand. Consequently, prices do not fall in a single direction but form a cycle of repeated rises and falls.

Although the term 're-contract' is not explicitly used in the text, the content describes a process of reconfiguring contractual relationships. Initially, each monopolist sets a monopoly price OP for their respective half of the market's customers. One monopolist lowers the price slightly to attract the rival's customers, and the other responds by lowering the price further. In this way, the price falls to OQ. However, due to supply constraints, neither monopolist can satisfy the entire demand, and some consumers are left unsatisfied. Consequently, , as one monopolist now stands to gain by supplying the residual demand at a higher price, the price rises again, and a new cycle begins.

Consequently, this process can essentially be understood as a process of re-contracting, in which consumers continually switch their contractual relationships to the more favourable seller.

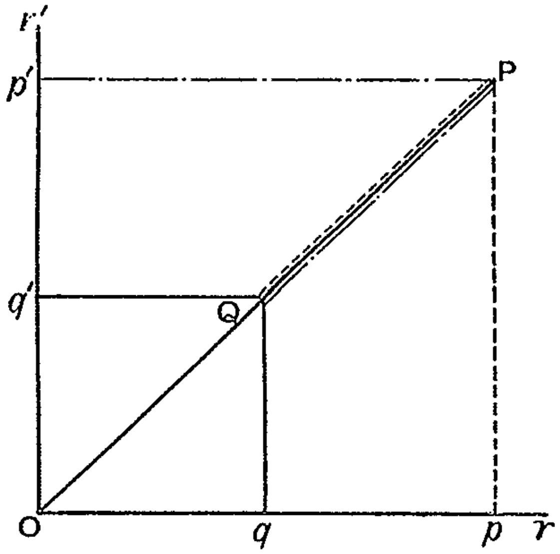

FIG. 2.

3-3 Interplay between complementary goods: the Nansen and Johansen analogy

Fig. 3 deals with the case of two monopolists selling complementary goods. Whereas Fig. 2 depicted the instability of identical and substitute goods, Fig. 3 addresses the question of 'what happens in the case of complementary goods'. The background here is the problem of two goods used together in a fixed ratio. In the case of complementary goods (pp. 124–126, Fig. 3), an equilibrium point P appears to exist at first glance. However, Edgeworth demonstrates that this is not a stable equilibrium.

The two sloping lines in the figure represent the paths of optimal prices for each monopolist. Price combinations that satisfy the profit-maximisation condition for each monopolist exist along these respective lines. The point of intersection P appears, at first glance, to be an equilibrium point. This is because, at that point, it seems that 'one party does not wish to move left or right, whilst the other does not wish to move up or down'. This is close to the Cournotian understanding of equilibrium. However, Edgeworth argues that P is not a true stable point. If one monopolist were to move even slightly, the other would react and move in turn, resulting in a further profit-generating move for the first monopolist. Consequently, the system could shift from P to another point, such as Q.

In the case of complementary goods, the prices set by each agent jointly determine demand. Consequently, a price change by one party does not merely alter the other's quantity demanded; it changes the very conditions under which the other's profit is maximised. What occurs here is a 'mutual transformation of optimal conditions', in the sense that one agent's attempt to improve their own profit alters the other's profit-maximising conditions, thereby changing the direction of the other's optimal response. Consequently, competition in the case of complementary goods is understood not as a struggle for demand , but as a dynamic in which each agent endogenously transforms the other's potential field.

In this sense, the Edgeworth cycle is not merely a phenomenon indicating the absence of equilibrium. It is a process in motion arising from the fact that, when potential is not a fixed topography but a field constantly transformed by interactions between agents, differences in marginal valuation continuously generate new directions of action. Consequently, competition should be understood not as a convergence towards a static equilibrium point, but as a process in motion unfolding under the interaction of field structure and potential.

In the main text, this process is explained using the metaphor of the Arctic explorers Nansen and Johansen. When rowing a sledge together across the Arctic plains, Nansen seeks to move towards the high points of the mountains, whilst Johansen seeks to move towards the deep points of the valleys. Each seeks to maximise their own objective function, but the optimal direction itself changes in response to the other's movement. Consequently, a chain reaction ensues whereby if one moves, the other moves too, and this in turn alters the optimal conditions for the first agent. What this metaphor illustrates is that even in the case of complementary goods, the point of intersection does not automatically constitute a stable equilibrium. Although this is not as obvious as in the case of substitute goods, as long as strategic reading of one another and interdependence exist, equilibrium can remain indeterminate here too. In other words, even in the case of complementary goods, interdependent movement persists.

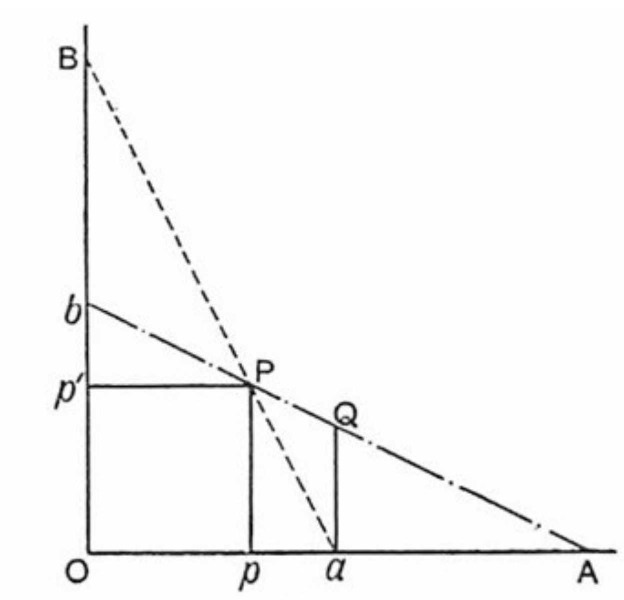

FIG. 3.

3-4 Relationship with Edgeworth (1881): Inheritance and Modification of the Re-contracting Process

Edgeworth's (1925) theory is closely related to the re-contracting theory in *Mathematical Psychics* (1881). The price movements of 1925 can be understood as the movements of the re-contracting process in 1881. This is because the competitive process described in terms of price movements in the 1925 text is translated into the language of the 1881 re-contracting theory in Note 2 on p. 118.

> "Mathematical Psychics. Competition between the two monopolists will reduce the price below the point Q in the figure on p. 114 (op. cit.) to within the range between Q and T. See the note on p. 110, where the statement that 'the system will reach a final settlement at some intermediate point' is inaccurate. Suppose that there are two D's dealing with an indefinite number of A's, as in the case now under consideration. The D's will force each other below point Q; and between that point and T the position of (temporary) equilibrium will continue to vary; since it will always be in the interest of one or more of the A's to re-contract with one or both of the D's; moving onto the partial or 'supplementary contract curves' which are indicated on p. 37 (op. cit.), but not represented in the figure on p. 114." (Edgeworth (1925) p. 118)

In 1881, Edgeworth defined competition as the 'possibility of re-contracting' and referred to a state that could not be improved upon by re-contracting as a 'final settlement'. However, in a note on p. 118, he explicitly corrected his 1881 assertion that 'the system reaches a final settlement at the midpoint between Q and T', labelling it 'inaccurate'. Instead, he argues that equilibrium is only temporary because the possibility of renegotiation along supplementary contract curves always remains.

The reason for this is that, when the effect of partial / supplementary contract curves—which had been overlooked in 1881—is applied to the case of two monopolists and many buyers, the system does not come to rest at a single point between Q and T. In a note from 1925, he states that when two Ds—that is, two monopolists—trade with a multitude of As, competition between the Ds pushes the price below Q, after which the position of the temporary equilibrium continues to shift between Q and T. The reason for this is that 'there is always at least one A who stands to gain from renegotiating with one or both Ds'. Furthermore, this dynamic is positioned as following the 'partial or supplementary contract curves' described in *Mathematical Psychics* p. 37.

This amendment is not merely a minor correction. In the 1881 view, a given point q between Q and T was a 'final settlement' that could no longer be altered by renegotiation. However, in the 1925 version, a point between the same Q and T— —becomes a 'temporary equilibrium' that holds only for the moment but is soon shifted by another renegotiation. This is because, when two monopolists deal with a large number of consumers, regardless of the price level, there remains an incentive for some consumers to 'renegotiate with the other monopolist'. Edgeworth explains in his 1925 work that this renegotiation takes place along the 'supplementary contract curves' outlined on p. 37 of the 1881 text. The 1925 revision applied this concept of supplementary contract curves to a market with two monopolists and many consumers, meaning that the point between Q and T is no longer a final settlement but a point constantly shifted by supplementary renegotiations. In other words, it can be interpreted that in 1925, Edgeworth himself further refined the logic of the renegotiation process that was inherent in the 1881 theory. Consequently, the 1925 cycle should not be understood as a break with the 1881 theory, but rather as the result of further refining the logic of the renegotiation process.

3-5 Relationship with Cournot and Bertrand

This theory of two monopolists by Edgeworth (1925) is fundamentally different from the contrast between Cournot (quantity competition) and Bertrand (price competition), which was later systematised in textbooks. In the Cournot model, firms choose quantities, and a Nash equilibrium is established at the intersection of their reaction functions. In the Bertrand model, price competition causes prices to fall to marginal cost.

However, by introducing supply constraints, interdependence and the possibility of renegotiation, Edgeworth reaches conclusions that differ from these. Of particular importance is the fact that he demonstrated that even in Bertrand-type price competition, there are cases where prices do not converge to an equilibrium. As Magnan de Bornier (1992) points out, Edgeworth did not merely inherit Bertrand's argument, but utilised it to reveal the limitations of the Cournotian concept of equilibrium.

3-6 Competition as Movement and Field

As is clear from the above analysis, competition in Edgeworth (1925) is an analysis of motion rather than an analysis of equilibrium points.

Each firm's profit depends on the prices set by others; consequently, at each point, the profit gradient—that is, the direction of 'which way to move for greater advantage'—changes. Agents adjust their prices in response to these local profit differentials, but since this adjustment itself alters the profit conditions of their competitors, the direction of movement is constantly updated. As a result, the system does not converge to a stationary point but continues to evolve.

This structure is analogous to the 'field' in physics. That is to say, whilst the direction of movement is determined at each point, that direction itself changes due to interactions, so that the system as a whole does not converge to a fixed point. In this sense, competition can be understood as 'the market as a vector field'. Rather than moving towards a fixed equilibrium point, each agent moves in accordance with local profit gradients. Furthermore, as this movement alters the conditions for other agents, a new vector field is formed . Consequently, Edgeworth's theory is based on a framework distinct from later equilibrium theories. Equilibrium is not the starting point of the analysis, but rather merely a phenomenon that appears temporarily within the course of movement.

3-7 Summary

Edgeworth (1925) demonstrated that prices do not converge under multiple monopolies and viewed competition as a process in motion. This theory builds upon the 1881 theory of re-contracting, taking it a step further by treating it as a process of movement. Furthermore, unlike the equilibrium theories of Cournot and Bertrand, it is characterised by understanding competition not as the 'establishment of equilibrium' but as the 'process in motion in which equilibrium does not arise'.

In this sense, Edgeworth's theory belongs to a different lineage from modern equilibrium-centred analysis; namely, it is situated within a theoretical tradition that understands competition as a process in motion.

4. Competition is a 'field'

As we saw in the previous section, Edgeworth's 1881 theory of contract and his 1925 theory of price competition differ in both their subject matter and their form of expression. The former deals with the agreement and renegotiation of exchange agents, whilst the latter deals with the price adjustments and cycles of oligopolistic firms. At first glance, the two appear to belong to distinct problems. However, if we focus on how competition is understood as a phenomenon, the two are linked by a single shared conceptual framework. Namely, the perspective that views competition not as the determination of an equilibrium point, but as a set of directions of change determined for each state—that is, as movement within a 'field'.

In the *Mathematical Psychics* (1881), the contract curve has often been understood as a set of efficient allocations. Certainly, such an interpretation is possible. However, what was significant in Edgeworth's own analysis was that the curve functioned not as a set of outcomes, but as a pathway along which the movement of exchange could proceed as long as the possibility of renegotiation persisted. Exchange does not come to rest at a single point; rather, as long as the possibility of renegotiation remains, it continues to move along the curve or in its vicinity. Here, competition appears not as a problem of choosing a final point, but as a movement from one distributional state to another.

This framework took on a more temporal and explicit form in 1925. In price competition, firms' behaviour differs according to each price state, and as a result, prices adjust in a certain direction. However, this adjustment is not a uniform convergence to equilibrium. Whilst Bertrand-style price cuts occur when prices are converging, the direction reverses upon reaching supply constraints, and the market shifts to a different domain in response to rival firms' reactions. Such competition is not a process of absorption into a single equilibrium point, but rather a movement proceeding under interdependence within the constraints of a domain that is permitted movable, giving rise to reversals at certain points of a boundary.

The key to a unified understanding of Edgeworth (1881) and Edgeworth (1925 [1897]) lies in where one places the 'basic unit of competition'. Within an equilibrium framework, the analysis of competition is typically constructed around the point where the optimal reactions of the agents converge, that is, the equilibrium point. In contrast, in Edgeworth's work, the analysis of competition is constructed from the perspective of the direction of movement—that is, which direction of change occurs in each state. The issue is not so much what the agents choose, but rather how the state changes under the relationships and constraints between agents. In this sense, competition is described not as a 'search for equilibrium' but as 'movement within a field'.

The 'field' referred to here denotes a structure in which the direction of change is determined at every point in space, just as in physics. However, the field in economics is not given as a natural law, but is constituted by profit, utility, constraints and interdependence. Consequently, it is not merely a metaphor, but signifies a cognitive choice regarding the unit in which competition is to be understood. For Edgeworth, competition does not appear as the sum of static choices made by agents, but as a field of motion under conditions of interaction.

Viewed in this way, the continuity between 1881 and 1925 becomes clear. In 1881, it was demonstrated that exchange does not cease through re-contracting—that is, the indeterminacy of contracts was shown. In 1925, it was demonstrated that price competition does not cease due to supply constraints and interdependence, thereby triggering cycles. In the former, the contract curve is the pathway of motion; in the latter, the cyclical trajectory on the price plane is the form of motion. What both have in common is that they understand competition not by the existence of an equilibrium point, but by the existence of continual motion.

More importantly, this 'field' perspective alters the very nature of the comparative static view. In equilibrium theory, an increase in the number of agents, a change in supply conditions, or the emergence of outside options is typically interpreted as a shift to a new equilibrium. However, from the field perspective, these are first and foremost conditions that transform the flow of competition. If the number of agents increases, the degree of freedom in renegotiation changes; if supply constraints tighten, the inflection points of the dynamic shift; and if outside options emerge, the boundary conditions of the field itself change. The question here is not 'where the equilibrium shifts to', but 'how the flow of competition is distorted'. Consequently, in Edgeworth's theory, the analysis of competition should be understood not as comparative statics, but as a structural change in the dynamic process.

In light of the above, the works of 1881 and 1925 can be positioned not merely as two distinct contributions addressing different problems, but as two developments of a single theoretical intuition: that 'competition is a field'. The 1881 work demonstrated this intuition in the space of exchange, whilst the 1925 work demonstrated it in the temporal dynamics of price competition. In other words, the 1881 work presented the geometry of the competitive field, whilst the 1925 work demonstrated the persistence of movement within the field. In this sense, Edgeworth's theory of competition should be re-evaluated not as an incomplete precursor to equilibrium theory, but as a unique theory that conceives of competition as a dynamic process in motion.

5. Maskin and Tirole (1988) and the Edgeworth cycle: the cycle as an equilibrium path and the cycle as a field of competition

Maskin and Tirole's (1988) theory of the Edgeworth cycle can be positioned as a reconfiguration of Edgeworth's critique of Bertrand within the framework of modern dynamic game theory. However, this relationship of inheritance is not straightforward. Maskin and Tirole inherit Edgeworth's insight that 'prices do not settle at a fixed equilibrium point', but they formulated this not as Edgeworth's own competitive process, but as an equilibrium path described by Markovian perfect equilibrium. It is in this respect— —that the historical continuity and discontinuity between the two schools of thought are simultaneously evident.

The paper referred to by Maskin and Tirole (1988) is "The Pure Theory of Monopoly", included in **Papers Relating to Political Economy**, vol. I (Edgeworth, 1925)[9] .In the introduction to MT, the concept of the Edgeworth cycle is described as 'originating with Edgeworth (1925)'; it is explained, in the context of the critique of Bertrand, that Edgeworth demonstrated that a static price equilibrium generally does not exist when firms face capacity constraints (Maskin and Tirole 1988, pp. 571–572).

The issue in Edgeworth (1925 [1897]) was that even when firms supplying homogeneous goods set prices, Bertrand-type conclusions do not hold if each firm faces capacity constraints. In the standard Bertrand model, since one firm can capture the entire market by setting a price slightly lower than its rival, prices are driven down to marginal cost. However, when capacity constraints are present, even a lower price cannot satisfy the entire market demand. Consequently, residual demand remains for the firm charging the higher price, leading to a conflict between the incentive to maintain the higher price and the incentive to lower the price in order to undercut the rival. From this structure, Edgeworth deduced the possibility that there might be no 'fixed' static equilibrium in price competition.

However, Edgeworth himself did not formulate this instability as a dynamic game in the modern sense. What he demonstrated was the logic that, under capacity constraints, the incentives to lower prices and the incentives to raise prices alternate from one phase to the next, meaning that a static equilibrium point can no longer govern the competition. Although an interpretation of prices as cyclical arises naturally from this, Edgeworth's analysis does not include concepts of dynamic games such as state variables, strategies, discount factors or equilibrium paths. Consequently, the 'cycle' in Edgeworth's work is, rather than a strict equilibrium path, an expression of competitive movement

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[9] The paper was originally published in the Economic Journal in 1897 and is included in the 1925 edition as 'E. The Pure Theory of Monopoly' on pp. 111–142 of Volume I. Consequently, in this paper, it is cited as Edgeworth (1925 [1897]).

arising from the absence of a static equilibrium.

In contrast, Maskin and Tirole (1988) formulated the cyclical price movements derived from Edgeworth as a dynamic game over an infinite period. In their model, firms take turns selecting prices, and those prices are committed to for a short period. Each firm's strategy is a Markov strategy that depends on the state—namely, the price set by the other firm immediately prior—and the concept of equilibrium is Markov-perfect equilibrium. MT demonstrated that, within this framework, both a kinked demand curve-type equilibrium, in which prices converge to a single point, and an Edgeworth cycle-type equilibrium, in which prices do not settle but oscillate, can exist. They describe the Edgeworth cycle as a cycle comprising a 'price war phase', in which firms compete to gain market share by cutting prices, and a 'reconciliation phase', in which one firm raises prices when the price war becomes sufficiently disadvantageous, and the other follows suit. Here, the cycle emerges not as a result of disequilibrium adjustment or exogenous disturbances, but as a consequence of rational equilibrium strategies.

What is particularly important in this reformulation is that, unlike the basic price-competition model of Maskin and Tirole (1988), it does not rely on the capacity constraints used by Edgeworth. Maskin and Tirole (1988) also state explicitly in a footnote that 'unlike in Edgeworth's treatment, the price cycle in this model does not depend on capacity constraints' (Maskin and Tirole 1988, p. 573 n. 3).In the model of Maskin and Tirole (1988), it is not physical constraints on supply capacity that generate the cycle, but rather short-term commitments, alternating turns, and strategic reactions that take future profits into account. In this respect, MT did not simply mathematise Edgeworth's phenomenon, but rather equilibrated Edgeworth-like instability through a different dynamic mechanism.

From this, the difference in their understanding of competition becomes apparent. In Maskin and Tirole (1988), competition is understood as a system of strategies. Firms choose the optimal response in each state, and equilibrium strategies are defined by the mutual consistency of these optimal responses. Therefore, even when cycles arise, this is not a failure of equilibrium but rather the temporal unfolding of equilibrium itself. The dynamics are organised by equilibrium, and price paths are derived from equilibrium strategies.

In contrast, in Edgeworth (1925), competition first appears as a local distribution of forces within the price space. That is to say, conditions such as the opponent's price, supply capacity, residual demand and the possibility of renegotiation create incentives to lower or raise prices at each stage. Price movements arise as a result of this, rather than a system of equilibrium strategies being given in advance. In this sense, Edgeworth's theory of competition is not one in which 'equilibrium explains movement', but rather one in which 'the structure of movement indicates the absence of equilibrium'.

In the terminology of this paper, whilst Maskin and Tirole (1988) views competition as a path to equilibrium, Edgeworth's approach contains an element of viewing competition as the movement of the 'competitive field'. The competitive field is the local configuration of forces that guides firms' behaviour at each price state. Edgeworth's capacity constraints act as conditions that distort this field. In the absence of constraints, price reductions would proceed in a Bertrandian manner towards marginal cost; however, when constraints are present, this movement reverses at a certain point, and an incentive for price increases emerges. What is important here is not where the new equilibrium point lies, but where, why, and in which direction the flow of competition turns.

It is therefore insufficient to say merely that Maskin and Tirole (1988) 'formalised' Edgeworth. More accurately, Maskin and Tirole (1988) reformulated the cyclical price phenomenon derived from Edgeworth within the framework of modern game theory's concept of equilibrium. This reformulation is of great significance in that it made it possible to analyse price cycles as rational equilibrium behaviour. On the other hand, however, Edgeworth's concept of a 'field in which competition moves without the equilibrium point' is reduced to a form of equilibrium strategy within the MT framework.

This difference relates to how competition is understood as an object. In Maskin and Tirole (1988), fluctuations in competition are situated within the concept of equilibrium. The cycle is the result of equilibrium strategies unfolding over time, and the focus of the analysis lies in the consistency of strategies. In contrast, in Edgeworth (1925), fluctuations in competition indicate that the equilibrium point cannot sufficiently organise competition. The focus of the analysis lies not in the equilibrium point, but in the direction of incentives, which changes with each price state, and their reversal.

From the above, the relationship between Edgeworth (1925) and Maskin and Tirole (1988) is not merely one of precursor and perfecter. Whilst both address the issue of prices being capable of fluctuating persistently, the former views this as the dynamics of the competitive field, whilst the latter views it as an equilibrium path. What lies beneath the same name, the 'Edgeworth cycle', is, on the one hand, a competitive process arising from the absence of equilibrium and, on the other, a price cycle constructed as a Markovian perfect equilibrium. By clarifying this distinction, Edgeworth's theory can be re-evaluated not as an incomplete precursor to modern dynamic game theory, but as a distinct theoretical possibility that understands competition as a process rather than an equilibrium.

6. Armstrong–Vickers and Edgeworth: From Equilibrium Theory to Movement Theory

In this section, we examine the reconstruction of the Edgeworth paradox by Armstrong and Vickers (2018, 2023) and situate it in relation to Edgeworth's (1925) understanding of competition. In their general theory of pricing for multi-product firms, Armstrong and Vickers (2018) treated the Edgeworth paradox—the possibility that prices may fall despite taxation or rising costs—as an implication of comparative statics. In contrast, Armstrong and Vickers (2023) placed the Edgeworth paradox itself at the centre of their analysis, examining the structure of cost pass-through in multi-product firms. The aim of this section is to clarify how the transition from the 2018 paper to the 2023 paper approaches the problem formulation of Edgeworth (1925), whilst also identifying where it still diverges from Edgeworth's 'perspective of viewing competition as a process of motion'.

6-1 Armstrong–Vickers (2018): Multi-product pricing as a comparative statics of equilibrium

The aim of Armstrong and Vickers (2018) was not to focus on the Edgeworth paradox itself, but rather to organise multi-product pricing in a general and concise manner. By expressing consumer surplus as a function of quantity rather than price, they treat multi-product pricing in monopoly, regulated monopoly and Cournot-type oligopoly in a unified manner. At the beginning of the paper, it is stated that 'profit-maximising and other Ramsey prices' and 'symmetric Cournot equilibrium' can be expressed using the gradient of the consumer surplus function (Armstrong and Vickers 2018, pp. 1444–1445).

The fundamental focus of this paper is how the equilibrium or optimal point changes in response to variations in parameters. For example, in the Ramsey problem, they examine how the optimal quantity and price-cost margin change when the weight a assigned to consumer surplus varies. Indeed, using first-order conditions, they express the deviation of the optimal price from marginal cost in the Ramsey problem in terms of the gradient of the consumer surplus function(Armstrong and Vickers 2018, pp. 1449–1450). Furthermore, regarding Cournot oligopoly, they demonstrate that the symmetric Cournot equilibrium corresponds to an appropriate Ramsey optimisation problem (Armstrong and Vickers 2018, pp. 1451–1453).

Consequently, the analysis in AV2018 is comparative statics based on equilibrium and optimal points. In other words, it examines in which direction an existing equilibrium point shifts when a certain parameter changes. This does not ask 'how the market moves over time', but rather 'how the point satisfying the equilibrium conditions changes in response to exogenous changes'. Within this framework, the Edgeworth paradox is positioned as a paradoxical result of comparative statics in multi-product pricing.

6-2 What is pass-through?

Here, we wish to clarify the meaning of 'pass-through', a central concept in the 2023 paper. Pass-through is a concept that describes how changes in costs or taxes are passed on to prices. For a single product, it asks how much the price rises when marginal cost increases by 1 yen. Normally, it is assumed that a rise in costs leads to a rise in prices.

However, the situation is different for multi-product firms. When the cost of a particular product rises, not only does the price of that product change, but the prices of other products may also change simultaneously. Consequently, pass-through is not merely a single ratio, but a matrix that illustrates how changes in the cost vector are transmitted to the entire price vector. In this sense, pass-through analysis is a form of comparative statics, but it explicitly addresses the interdependence of the entire price system more than standard comparative statics does. In other words, whereas comparative statics asks 'how does the equilibrium point change?', pass-through analysis asks 'how do cost changes propagate through the price system?'.

6-3 Armstrong–Vickers (2023): The Edgeworth paradox as a structural constraint on pass-through

Armstrong and Vickers (2023) is a paper that directly links this pass-through problem to the Edgeworth paradox. At the beginning of the paper, they define Edgeworth's paradox of taxation as 'the phenomenon whereby a multi-product monopolist lowers prices when the unit cost or tax on a particular product rises' (Armstrong and Vickers 2023, p. 2645). Furthermore, in the introduction, after confirming that Edgeworth presented this paradox in his 1897 paper and that it was included in the 1925 English edition, they specifically refer to the two-goods case in Edgeworth (1925, pp. 132–134). There, an example is given where a tax on the first good can cause the price of not only the first good but also the second good to fall(Armstrong and Vickers 2023, pp. 2646–2647).

The key point of the 2023 paper is that it does not treat the Edgeworth paradox as merely a curious example, but rather formulates it as a general problem: 'What kind of pass-through matrix is consistent with standard profit maximisation?'.They state that 'in a multi-product firm, there are n × n pass-through terms' and ask what constraints these terms face under standard cost and demand theory (Armstrong and Vickers 2023, p. 2647).

This point becomes clearer at the beginning of Section III. They state that, in order to analyse the possibility of the Edgeworth paradox in general, they will study 'feasible patterns of cost pass-through' (Armstrong and Vickers 2023, p.2651). In the case of linear demand and linear marginal cost, the optimal quantity is derived from the first-order condition of profit maximisation, and this demonstrates how the optimal price reacts to costs. They define Γ as the 'matrix derivative dp/dc' and explicitly state that this is the cost pass-through matrix for price (Armstrong and Vickers 2023, p.2652).

Furthermore, they demonstrate that not just any pass-through matrix is possible; rather, for it to be consistent with profit maximisation, it must satisfy certain structural constraints. In their general analysis, they show that the condition for Γ to be a feasible price-cost pass-through matrix is that it must be 'similar to a positive-definite matrix', i.e. a diagonalizable matrix with positive eigenvalues (Armstrong and Vickers (2023), p.2658).

Consequently, Armstrong and Vickers (2023) does not merely remain within the realm of comparative statics, which asks whether 'prices rise or fall when taxes increase'. Rather, it analyses which directions of price reaction are theoretically permissible whilst maintaining the equilibrium conditions. In other words, Armstrong and Vickers (2023) conceives of pass-through as a structure of 'possible directions of movement' in the vicinity of the equilibrium point.

6.4 Why AV2023 is moving closer to Edgeworth (1925)

In this respect, Armstrong and Vickers (2023) comes quite close to the ideas of Edgeworth (1925).

What was important in Edgeworth (1925) was that the price system does not simply converge to a single equilibrium point; rather, because the profit conditions of each agent are interdependent, the direction of behaviour at one point changes at another. At the current price level, firms determine which direction to move prices in order to increase profits. However, as this action alters the profit conditions of other firms, a different direction of movement arises at the next time step.

Armstrong and Vickers (2023) also addresses the issue of interdependent local directions within the price system.The pass-through matrix indicates which direction the price vector should move in to be consistent with profit maximisation when costs change at a given equilibrium point. Therefore, viewed locally from any equilibrium point, the ‘permissible directions of price reaction’ analysed by Armstrong and Vickers (2023) overlap with the ‘locally possible directions of movement’ in Edgeworthian price movements.

However, this overlap is strictly local. In Edgeworth’s framework, local directions generate actual movements. When one agent moves, the conditions for the other agent change as a result, giving rise to further movement in a new direction. In other words, what Edgeworth is concerned with are trajectories in price space.

In contrast, in Armstrong and Vickers (2023), the equilibrium conditions are maintained. What they analyse is not how a market that has deviated from the equilibrium point adjusts, but rather in which direction a new equilibrium price lies that satisfies the first-order condition of profit maximisation even after a change in costs. Therefore, pass-through in Armstrong and Vickers (2023) is not an analysis of the movement itself, but rather of the ‘directions in which movement is possible whilst maintaining equilibrium’.

This difference can be summarised as follows: Armstrong and Vickers (2018): How does the equilibrium point change? Armstrong and Vickers (2023): In which direction can the price system move whilst maintaining equilibrium? Edgeworth (1925): Not merely remaining at equilibrium, but what trajectory does the competitive process itself follow?

Consequently, whilst Armstrong and Vickers (2023) approaches Edgeworth’s problem formulation, it still interprets it within the framework of equilibrium theory . In other words, Armstrong and Vickers (2023) is a theory that extracts the ‘local differential structure’ of Edgeworth’s movement, but it does not, like Edgeworth, view competition as a temporal contract movement.

6.5 Why is the Edgeworthian approach important today?

This distinction is particularly important when considering modern platform markets. Today’s large-scale platform firms do not merely set the price of a single product. They manage multiple market dimensions simultaneously, combining prices, fees, subsidies and algorithmic preferential treatment for different groups, such as users, advertisers, merchants and delivery personnel.

In such markets, the situation is not a simple one where a single cost change is passed on to a single price; rather, the entire pricing structure operates in an interdependent manner. Consequently, the analysis of the pass-through matrix in Armstrong and Vickers (2023) holds significant contemporary relevance. The structural constraints of pass-through provide a crucial clue when considering which market segments a platform passes costs onto and in which segments it conversely lowers prices.

However, in platform markets, this alone is not sufficient. This is because platform firms do not adjust prices just once in response to cost changes, but continuously adjust fees, recommendation algorithms, search rankings, bundling, loyalty schemes, and contractual terms with third-party providers. Furthermore, the profit function itself changes due to user migration, multi-homing, network effects, and data accumulation.

In such markets, what matters is not merely 'where the equilibrium price lies'. Rather, the key issues are how market power is formed, through what trajectory; how exclusion occurs, and through what process; and how network advantages accumulate. These aspects are difficult to capture through the comparative statics of equilibrium points alone.

Here, the perspective of Edgeworth's contract dynamics regains significance. Edgeworth viewed competition not as a convergence towards a fixed equilibrium point, but as a process in motion in which each agent continually renegotiates contract terms in pursuit of local gains. If we develop this perspective in a contemporary context, we can analyse platform markets as a dynamic process in which prices and contracts are constantly being restructured.

Taking this a step further, it is also possible to view this contractual dynamics through the lens of the calculus of variations. That is to say, we take the market trajectory itself as the object of study and ask which market dynamics are socially desirable, and which should be controlled because they lead to exclusion or lock-in. Of course, this requires the explicit specification of state spaces, time structures, laws of motion, and action functions, necessitating stronger assumptions than those of equilibrium theory. However, this enables a more dynamic and normative policy analysis focused not on 'which equilibrium is efficient', but on 'which market dynamics should be permitted'.

Consequently, Armstrong and Vickers (2023) approaches the Edgeworthian direction, yet this approach remains confined within the framework of equilibrium theory. To understand contemporary platform markets, it is necessary to take this local pass-through analysis as a starting point and extend it to an analysis of Edgeworthian contractual movements and, furthermore, market trajectories. Herein lies the significance of a modern reinterpretation of Edgeworth's theory.

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The structure of Edgeworth (1881) is as follows.

Introductory Description of Contents, (pp. v–viii)

Mathematical Psychics may be divided into two parts.
Theoretical and Applied.

Part (I) (Theoretical)

- Mathematical reasoning without numerical data
- An analogy is suggested between the principles of greatest happiness, utilitarian or egoistic, which constitute the first principles of ethics and economics, and those principles of maximum energy which are among the highest generalisations of physics.

Part (II) (Applied) Calculus of Pleasure

- Economic Calculus
- Utilitarian Calculus

Appendices

I. On Non-numerical Mathematics
II. On the Importance of Hedonical Calculus
III. On Hedonimetry
IV. On Mixed Forms of Utilitarianism
V. On Professor Jevons's Formulas of Exchange
VI. On the Errors of TBE tiyrwparptlroi
VII. On the Current Crisis in Ireland

On the Application of Mathematics to the Moral Science (pp. 1–16)

Part I

Non-numerical Mathematics
Hedonimetry
Maximum Energy
Maximum Pleasure
Social Mechanics
Man as a Pleasure Machine

Part II

Economical Calculus (pp. 16–56)

Economic Definitions
Pure contract
Perfect Competition
Imperfect Competition
Perfect Competition
Combinations
Cooperative Associations
Need for Arbitration
Principle of Arbitration

Utilitarian Calculus (pp. 56–82)

Utilitarian Calculus
Definitions
The Unit of Pleasure
Law of Diminishing Utility
Distribution of Means
Distribution of Work
Utilitarian Selection
The Least Favoured Class
Proof of Utilitarianism
Application of Utilitarianism

Appendices (pp. 83–148)

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