『経済研究』（明治学院大学）第130号　2004年

Imperfect Competition and General Equilibrium: A Generalization of Negishi's Approach to Technologies with Fixed Costs

Pierre DEHEZ*
Jacques DREZE†
Takashi SUZUKI‡

1 Introduction

In the last fifty years, general equilibrium theory has established successful description of the perfectly competitive market which is culminated as so called Arrow-Debreu model (Arrow and Debreu (1954), McKenzie (1954), and Nikaido (1956)). In this theory, a large number of firms producing homogeneous goods act through a price system taken by each one of them as given. The mathematical foundations of this theory based on equilibrium existence theorems are solid and satisfactory.

On the other hand, Chamberlin (1956) and Robinson (1933) proposed theories of imperfect competition. Inspired by their work, several theoretical economists have sought to obtain consistent theories. In contrast with perfect competition, imperfectly competitive firms enjoy some degree of monopolistic power over the commodities they produce, and consequently, they do not take prices as given. An obvious reason for such monopolistic power is that the number of active firms in the market is small, possibly because large amount of the set-up cost is required for entering and operating in the market and/or technology exhibits increasing returns. Therefore one of the most significant task in order to construct a realistic model of the monopolistic market is to build up a theory incorporating the firms with nonconvex production sets. To our knowledge, no previous works of the field have achieved such a construction. This paper takes the first step toward this task.

In a classical paper of 1961, Negishi first succeeded to construct a general equilibrium model with firms who do not take as given, but rather they calculate their profit based on their expected

* Université Catholique de Louvain, College L. H. Dupriez, place Montesquier 3, B-1348 Louvain la Neuve, Belgium
† CORE, Université Catholique de Louvain, 34 Voie du Roman Pays, B-1348 Louvain la Neuve, Belgium
‡ Meiji-Gakuin University 1-2-37 Shiroganedai, Minoto-ku, Tokyo, 108, Japan

(perceived) demand functions. Unfortunately, Negishi assumed that each firm has convex production set. As Arrow and Hahn (1971) pointed out, this is a severe condition since the occurrence of monopoly is unlikely under those circumstances, as Negishi himself noted (p.109, middle).

In this note we extend his model to a case of nonconvex production sets which come from nontrivial set-up costs. In so doing we use heavily the previous work of the first two authors in the context of the theory of the competitive firms with nonconvex production sets. We believe that this extension is an important progress of general equilibrium theory. In the next section, we present the model and prove the main result.

2 The Consumers and the Producer

We assume that there are three kinds of commodities, indexed by h=1, 2, 3. The commodity 1 is a fixed input (see (1) below), the commodity 2 is a variable input and the commodity 3 is a consumption good (output). Then the commodity space is $\mathbb{R}^3$ and a typical commodity bundle is denoted by $= (x^1, x^2, x^3) \in \mathbb{R}^3$, and a price vector by $\boldsymbol{p} = (p^1, p^2, p^3) \in \mathbb{R}^3$.

There exist n consumers in the economy, indexed by $i = 1 \ldots n$. As usual, the consumer i is characterized by the consumption set X_i, the preference relation $\geq_i \subset X_i \times X_i$ which is a complete and transitive binary relation on X_i, the endowment vector $\omega_i = (\omega_i^1, \omega_i^2, \omega_i^3) \in \mathbb{R}^3$ and the firm share $\theta_i \geq 0$ such that $\sum_i \theta_i = 1$.

We assume that there exists only one firm (the monopolist) in the economy. The firm's production set is defined by

$$Y \doteq \{\boldsymbol{y} \in \mathbb{R}^2_- \times \mathbb{R}_+ \mid y^3 \leq F(y^2) \text{ if } y^1 \leq -c \text{ and } y^3 \leq 0 \text{ if } -c < y^1 \leq 0\} \tag{1}$$

where $c \geq 0$ is some constant. The production function $F\colon \mathbb{R}_- \to \mathbb{R}_+$ is assumed to be continuously differentiable, strictly decreasing and concave and to satisfy $F(0) = 0$. The set of weakly efficient production plans is by definition, $\partial Y = \{\boldsymbol{y} \in Y \mid \text{There exists no } \boldsymbol{y}' \in Y \text{ with } \boldsymbol{y}' \gg \boldsymbol{y}\}$ and the set of efficient production plan is $E' = E \cup \boldsymbol{0}$ where

$$E = \{\boldsymbol{y} \in Y \mid y^1 = -c, y^2 < 0, y^3 = F(y^2)\} \tag{2}$$

is the subset of weakly efficient plans involving a positive output ("active part" of the production process).

The corresponding cost function is given by

$$C(y^3 \mid p^1, p^2) = \begin{cases} p^1 c + p^2 G(y^3) & \text{if } y^3 > 0 \\ 0 & \text{if } y^3 \leq 0 \end{cases} \tag{3}$$

where $G(y^3) = -F^{-1}(y^3)$. Obviously the cost function is continuously differentiable, strictly increasing and convex function of output.

The marginal cost function

$$C'(y^3|p^2) = p^2G'(y^3) \tag{4}$$

is defined for $y^3 > 0$ and p^2. It is increasing in output and homogeneous of degree 1 in prices. The elasticity of the variable cost function is given by

$$\gamma(y^3) = \frac{y^3G'(y^3)}{G(y^3)}. \tag{5}$$

Convexity of the function G implies that $\gamma(y^3) \geq 1$ for all $y^3 \geq 0$. It is nothing but the ratio of the marginal variable cost over average variable cost.

3 Perceived Demand

The firm buys its input on competitive market but holds conjectures on the relation between the demand for its product price at which it is sold. This is an equilibrium observation in the sense that if the firm would produce and sell the quantity y^3, the market would be in equilibrium. The firm is assumed to have a local conjecture on the behavior of prices around the equilibrium point (y^3, p^3) and the conjecture may depend on the state of the economy represented by the excess demand vector $\boldsymbol{z} = (z^1, z^2, z^3)$ and the price system $\boldsymbol{p} = (p^1, p^2, p^3)$. That conjecture is given by the slope $b(\boldsymbol{z}, \boldsymbol{p})$ of some perceived demand curve $q^3 = f(y^3|\boldsymbol{z}, \boldsymbol{p})$ evaluated at $y^3 = z^3$, i.e.,

$$b(\boldsymbol{z}, \boldsymbol{p}) = f'(z^3|\boldsymbol{z}, \boldsymbol{p}), \tag{6}$$

where f' denotes the partial derivative of f with respect to y^3. Here $q^3 = f(y^3|\boldsymbol{z}, \boldsymbol{p})$ is the price at which the firm expects to sell the quantity y^3 when it observes $(\boldsymbol{z}, \boldsymbol{p})$. The perceived demand function is assumed to satisfy the following natural assumptions.

D.1 Consistency with observations:

$$f(z^3|\boldsymbol{z}, \boldsymbol{p}) = p^3 \text{ for all } \boldsymbol{z}, \boldsymbol{p},$$

D.2 Homogeneity with respect to observable prices:

$$f(y^3|\boldsymbol{z}, \lambda\boldsymbol{p}) = \lambda f(y^3|\boldsymbol{z}, \boldsymbol{p}) \text{ for all } \lambda > 0,\ y^3 > 0, z \text{ and } p,$$

D.3 Law of demand:

$$b(\boldsymbol{z}, \boldsymbol{p}) < 0 \text{ for all } \boldsymbol{z} \text{ and } \boldsymbol{p}.$$

Because we are only interested in local maxization of profit (see below), only a local knowledge of demand actually matters. We assume that the perceived demand is locally linear.

D.4 Local lineality:

$$f(y^3 | \boldsymbol{z}, \boldsymbol{p}) = \max\{0, a(\boldsymbol{z}, \boldsymbol{p}) + b(\boldsymbol{z}, \boldsymbol{p})y^3\} \text{ for all } y^3 \text{ in some neighborhood of } z^3.$$

Consistency D.1 implies that $a(\boldsymbol{z}, \boldsymbol{p}) + b(\boldsymbol{z}, \boldsymbol{p})z^3 = p^3$. Hence whenever $q^3 > 0$, the perceived demand function can be written as

$$q^3 = p^3 + b(\boldsymbol{z}, \boldsymbol{p})(y^3 - z^3), \tag{7}$$

that is, the differences in prices are proportional to the differences in quantities. The elasticity of the perceived demand around the point $(\boldsymbol{z}, \boldsymbol{p})$ is given by

$$\eta(\boldsymbol{z}, \boldsymbol{p}) = -\frac{z^3 f'(z^3 | \boldsymbol{z}, \boldsymbol{p})}{p^3} = -\frac{z^3 b(\boldsymbol{z}, \boldsymbol{p})}{p^3} > 0. \tag{8}$$

By the assumption of homogeneity, the slope is homogeneous of degree 1 in $\boldsymbol{p}$. As a consequence, the elasticity is homogeneous of degree 0 in $\boldsymbol{p}$. The next assumption.

D.5 Continuity:

$$b(\boldsymbol{z}, \boldsymbol{p}) \text{ is a continuous function at } (\boldsymbol{z}, \boldsymbol{p}) \in \mathbb{R}^3 \times \mathbb{R}_{++},$$

ensures the continuity of the function η at $(\boldsymbol{z}, \boldsymbol{p}) \in \mathbb{R}^3 \times \mathbb{R}_{++}$. Notice that we assume that the slope and the elasticity are defined for all $\boldsymbol{z} \in \mathbb{R}^3$ and all $\boldsymbol{p} \in \mathbb{R}^3_{++}$ and in particular $z^3 < 0$.

4 Equilibrium

The profit function can be written as a function of y^3, price vector $\boldsymbol{p}$ and the perceived excess demand $\boldsymbol{z}$,

$$\boldsymbol{\pi}(y^3 | \boldsymbol{z}, \boldsymbol{p}) = f(y^3 | \boldsymbol{z}, \boldsymbol{p})y^3 - p^2 G(y^3) - p^1 c = f(y^3 | \boldsymbol{z}, \boldsymbol{p})y^3 - C(y^3 | p^1, p^2). \tag{9}$$

For any given $(\boldsymbol{z}, \boldsymbol{p})$, the first order condition for a maximum of profit with respect to y^3 is given by

$$f(y^3 | \boldsymbol{z}, \boldsymbol{p}) + y^3 b(\boldsymbol{z}, \boldsymbol{p}) = p^2 G'(y^3), \tag{10}$$

assuming $y^3 > 0$. At an equilibrium $(y^3 = z^3)$ the usual first order condition applies,

$$p^3(1 - \eta(\boldsymbol{z}, \boldsymbol{p})) = C'(z^3 | p^2). \tag{11}$$

If the corresponding value of profit is nonnegative, we require that this is a necessary and sufficient condition for a local maximum. At one point, however, the condition is treated as exceptional. That is the local maximization at the origin, $\mathbf{0} \in \mathbb{R}^3$. This is a nondifferentiable point of the production set, and $\boldsymbol{\pi}(y^3 | \mathbf{0}, \boldsymbol{p}) \leq \boldsymbol{\pi}(0 | \mathbf{0}, \boldsymbol{p})$ for every nonnegative price vector $\boldsymbol{p}$ and all y^3 in a neighborhood of $\mathbf{0}$ in Y, since we have $y^3 \leq 0$ and $C(y^3 | p^1, p^2) = 0$ there. Thus we suppose that at the origin, the condition of local maximization is satisfied for every $\boldsymbol{p} \in \mathbb{R}^3_+$, although the differential form (11) does not hold. We do not care about the other nondifferential points, since they would not be equilibrium points.

An allocation is an $n+1$-tuple of commodity vectors, $\{(\boldsymbol{x}_i), \boldsymbol{y}\}_{i=1}^{n}$, where $\boldsymbol{x}_i$ and $\boldsymbol{y}$ is a consumption vector of the consumer i and the production vector of the monopolist, respectively. An allocation $\{(\boldsymbol{x}_i), \boldsymbol{y}\}$ is said to be feasible if $\sum_{i=1}^{n} \boldsymbol{x}_i = \sum_{i=1}^{n} \omega_i + \boldsymbol{y}$. The list $\varepsilon = (X_i, \succsim_i, \omega_i, \theta_i, f, F)$ is called an economy.

Now we state the definition of our equilibrium concept.

Definition: A price vector $\hat{\boldsymbol{p}}$ and an allocation $\{(\hat{\boldsymbol{x}}_i), \hat{\boldsymbol{y}}\}$ is called a monopolistic (ally competitive) equilibrium if the following conditions hold,

(E-1) $\hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_i = \hat{\boldsymbol{p}}\omega_i + \theta_i \hat{\boldsymbol{p}}\hat{\boldsymbol{y}}$ and if $\hat{\boldsymbol{p}}\boldsymbol{x} \leq \hat{\boldsymbol{p}}\omega_i + \theta_i \hat{\boldsymbol{p}}\hat{\boldsymbol{y}}$, then $\hat{\boldsymbol{x}}_i \succsim_i \boldsymbol{x}$ for all $\boldsymbol{x} \in X_i$, $i = 1, \ldots, n$,
(E-2) $\hat{p}^3(1-\eta(\hat{\boldsymbol{y}}, \hat{\boldsymbol{p}})) = C'(\hat{y}^3 | \hat{p}^2)$
(E-3) $\pi(\hat{y}^3 | \hat{\boldsymbol{y}}, \hat{\boldsymbol{p}}) \geq 0$,
(E-4) $\sum_{i=1}^{n} \boldsymbol{x}_i = \sum_{i=1}^{n} \omega_i + \hat{\boldsymbol{y}}$.

Remarks: The condition (E-1) is the standard utility-maximization condition. The conditions (E-2) and (E-3) are the condition of local profit maximization and the nonnegative profit condition, respectively. As we have explained above, we suppose that the condition (E-2) is automatically met at the origin for every nonnegative price vector. Of course the profit is zero at that point, so that the condition (E-3) is also met. The condition (E-4) is the market condition, which states that the demand is equal to supply.

Our main result reads;

Theorem: For the economy $\varepsilon = (X_i, \succsim_i, \omega_i, \theta_i, f, F)$, there exists a monopolistic (ally competitive) equilibrium if the following conditions hold for every $i (= 1, \ldots, n)$.

(C-1) X_i is a convex and closed subset of $\mathbb{R}^3$, which is bounded below,
(C-2) $\succsim_i$ is closed in $X_i \times X_i$ (continuity),
(C-3) for every $\boldsymbol{x}, \boldsymbol{z} \in X_i$ such that $\boldsymbol{x} \prec_i \boldsymbol{z}$ and every $0 < t < 1$, it follws that $\boldsymbol{x} \prec_i t\boldsymbol{x} + (1-t)\boldsymbol{z}$ (convexity)
(C-4) for every $\boldsymbol{x} \in X_i$, $\boldsymbol{x} \prec_i \boldsymbol{x} + \boldsymbol{z}$ for every $\boldsymbol{z} > 0$ (monotonicity),
(C-5) there exists $\bar{\boldsymbol{x}}_i \in X_i$ such that $\bar{\boldsymbol{x}}_i \ll \omega_i$ (adequate endowment).

Remark: It is easy to see that the production set Y possesses the following properties.

(Y-1) Y is a closed subset of $\mathbb{R}^3$,
(Y-2) $Y + \mathbb{R}^3_- \subset Y$,
(Y-3) $Y \cap \mathbb{R}^3_+ = \{0\}$,
(Y-4) for all $\boldsymbol{z} \in \mathbb{R}^3$, the set $\{\boldsymbol{y} \in Y | \boldsymbol{y} \geq \boldsymbol{z}\}$ is bounded in $\mathbb{R}^3$.

5 Proof

To prove the extence of an equilibrium, we construct a pricing rule defined by a correspondence $\hat{\phi}: \partial Y \times \mathbb{R}^3 \times \mathbb{R}^3_+ \to \mathbb{R}^3_+$. For technical reasons, it must be closed (upper hemi-continuous) and the value $\hat{\phi}(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p})$ are closed, convex and nondegenerate cones. As a consequence, when the value is intersected with the price simplex, it defines a correspondence which is upper hemi-continuous and have nonempty, compact and convex values.

Let us define the correspondence $\phi: \mathbb{R}_+ \times \mathbb{R}^3 \times \mathbb{R}^3_+ \to \mathbb{R}^3_+$ by

$$\phi(y^3\,|\,\boldsymbol{z}, \boldsymbol{p}) = \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^3(1-\eta(\boldsymbol{z}, \boldsymbol{p})) = q^2 G'(y^3) \text{ and } q^3 y^3 \geq q^1 c + q^2 G(y^3)\}. \tag{12}$$

It is the prices such that at an equilibrium ($\boldsymbol{q} = \boldsymbol{p}$ and $y^3 = z^3 > 0$), the first order condition is satisfied and profit is nonnegative. It thereby defines the set of prices such that, at an equilibrium, profit is locally maximum at y^3. Notice that it does not correspond exactly to this outside an equilibrium. Indeed, $f(y^3\,|\,\boldsymbol{z}, \boldsymbol{p})$ has been replaced by q^3 and η is evaluated at the point $(\boldsymbol{z}, \boldsymbol{p})$.

In the definition of ϕ, the equation imposes a condition on q^2/q^3 while the inequality imposes a condition on q^1/q^3. Indeed, extracting q^2 from the equation gives the following inequality:

$$q^3 y^3 \left(1 - \frac{1-\eta(\boldsymbol{z}, \boldsymbol{p})}{\gamma(y^3)}\right) \geq q^1 c. \tag{13}$$

The equation $q^3(1-\eta(\boldsymbol{z}, \boldsymbol{p})) = q^2 G'(y^3)$ is actually valid only if $1-\eta(\boldsymbol{z}, \boldsymbol{p}) > 0$. As a consequence, $\phi(y^3\,|\,\boldsymbol{z}, \boldsymbol{p})$ is a closed, convex and nondegenerate cone for all $y^3 > 0$ and all $(\boldsymbol{z}, \boldsymbol{p})$ such that $\eta(\boldsymbol{z}, \boldsymbol{p}) < 1$.

To define the pricing rule $\hat{\phi}$, it remains to complete ϕ "in a continuous way".

a. for all $\boldsymbol{y} \in E$,

$$\hat{\phi}(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p}) = \begin{cases} \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^3(1-\eta(\boldsymbol{z}, \boldsymbol{p})) = q^2 G'(y^3),\ q^3 y^3 \geq q^1 c + q^2 G(y^3)\} & \text{if } \eta(\boldsymbol{z}, \boldsymbol{p}) < 1, \\ \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^2 = 0,\ q^3 y^3 \geq q^1 c\} & \text{if } \eta(\boldsymbol{z}, \boldsymbol{p}) \geq 1, \end{cases}$$

b. for all $\boldsymbol{y}$ such that $y^1 < -c$, $y^2 < 0$ and $y^3 = F(y^2)$,

$$\hat{\phi}(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p}) = \begin{cases} \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^1 = 0,\ q^3(1-\eta(\boldsymbol{z}, \boldsymbol{p})) = q^2 G'(y^3),\ q^3 y^3 \geq^2 G(y^3)\} & \text{if } \eta(\boldsymbol{z}, \boldsymbol{p}) < 1, \\ \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^1 = q^2 = 0\} & \text{if } \eta(\boldsymbol{z}, \boldsymbol{p}) \geq 1, \end{cases}$$

c. for all $\boldsymbol{y}$ such that $y^1 = -c$, $y^2 < 0$ and $0 \leq y^3 < F(y^2)$,

$$\hat{\phi}(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p}) = \{q \in \mathbb{R}^3_+ \,|\, q^1 = 0,\ q^3 y^3 \geq -q^2 y^2\},$$

d. for all $\boldsymbol{y}$ such that $-c < y^1 < 0$, $y^2 < 0$ and $y^3 = 0$,

$$\hat{\phi}(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p}) = \{q \in \mathbb{R}^3_+ \,|\, q^1 = q^2 = 0\},$$

e. for all $\boldsymbol{y} \in \partial Y$ along the axis,

$$\hat{\phi}(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p}) = \begin{cases} \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^1 = 0\} & \text{if } y^1 < 0,\ y^2 = y^3 = 0, \\ \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^2 = 0\} & \textit{if } y^2 < 0,\ y^1 = y^3 = 0, \\ \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^3 = 0\} & \textit{if } y^3 < 0,\ y^1 = y^2 = 0, \end{cases}$$

f. for all $\boldsymbol{y} \in \partial Y$ on the negative faces,

$$\hat{\phi}(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p}) = \begin{cases} \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^2 = q^3 = 0\} & \text{if } y^1 = 0,\ y^2 < 0,\ y^3 < 0, \\ \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, q^1 = q^3 = 0\} & \text{if } y^2 = 0,\ y^1 < 0,\ y^3 < 0, \end{cases}$$

g. at the origin,

$$\hat{\phi}(0\,|\,\boldsymbol{z}, \boldsymbol{p}) = \mathbb{R}^3_+.$$

Notice that in cases (d) to (g), $\phi(\boldsymbol{y}\,|\,\boldsymbol{z}, \boldsymbol{p})$ is just Clark's normal cone.

Proof of the theorem: Under the conditions (C-1), (Y-3) and (Y-4), it is well known that the set of feasible allocations,

$$A = \left\{(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n, \boldsymbol{y}) \in \prod_i X_i \times Y \,\middle|\, \sum_i \boldsymbol{x}_i = \sum_i \omega_i + \boldsymbol{y}\right\}. \tag{14}$$

is nonempty and bounded. Thererore there exists a closed cube K in $\mathbb{R}^3$ with length $k > 0$ and centered at the origin such that if $((\boldsymbol{x}_i), \boldsymbol{y}) \in A$, then $x_i, y \in$ *interior* K, 1, ..., n. Recall that ∂Y is the set of weakly efficient production vectors (see (3)). Using this notation, set $\bar{Y} = \partial(Y + k\mathbf{1}) \cap \mathbb{R}^3_+$, where $k > 0$ is the length of the set K and $\mathbf{1} = (1, 1, 1,)$. Define $\hat{Y} = \{\boldsymbol{y} = (y^1, y^2, y^3) \in \bar{Y} \,|$ There exists no $\boldsymbol{z} = (z^1, z^2, z^3) \in \bar{Y}$ with $\boldsymbol{z} \geq \boldsymbol{y}$ such that $z^3 > y^3$ if $y^3 > 0\}$, and $\hat{X}_i = X_i \cap K$. Let ν denote the projection of points in $\mathbb{R}^3_+ - \{0\}$ on the unit simplex $S = \{\boldsymbol{q} \in \mathbb{R}^3_+ \,|\, \sum_{h=1}^{3} q^h = 1\}$. Then Brown et al. (1986) proved that under (Y-1) to (Y-4), the map ν: $\hat{Y} \rightarrow S$ is a homoeomorphism between $\hat{Y}$ and S which satisfies

$$\nu(\boldsymbol{y}) \gg 0 \text{ if and only if } \boldsymbol{y} \gg 0. \tag{15}$$

Define the function g on S by $g(\boldsymbol{q}) = \nu^{-1}(\boldsymbol{q}) - k\mathbf{1}$. By the assumption (Y-2), it follows that $g(\boldsymbol{q}) \in \partial Y$ for every $\boldsymbol{q} \in S$. Let the quasi-demand correspondence of the consumer i, ξ_i: $S^2 \rightarrow X_i$ be defined by

$$\xi_i(\boldsymbol{p}, \boldsymbol{q}) = \begin{cases} \{\boldsymbol{x} \in X_i \,|\, \boldsymbol{px} = \boldsymbol{p}\omega_i + \theta_i \boldsymbol{p}q(\boldsymbol{q}), \text{ and} & \\ \boldsymbol{pz} > \boldsymbol{p}\omega_i + \theta_i \boldsymbol{p}g(\boldsymbol{g}) \text{ whenever } \boldsymbol{x} <_i \boldsymbol{z}\} & \text{if } \boldsymbol{p}\omega_i + \theta_i \boldsymbol{p}g(\boldsymbol{q}) > \text{in } f\boldsymbol{p}X_i \\ \{\boldsymbol{x} \in X_i \,|\, \boldsymbol{px} = \text{in } f\boldsymbol{p}X_i\}, & \text{otherwise.} \end{cases} \tag{16}$$

It is well known that the quasi-demand correspondence is upper hemi-continuous and has nonempty, compact convex values (Debreu (1962)).

The supply correspondence of the firm is a function β: $S^3 \rightarrow S$ defined by

$$\beta(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}) = \frac{\alpha(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r})}{\lambda(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r})}, \tag{17}$$

where $\alpha(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}) = (\alpha^h(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r})) = (\max\{0, p^h + q^h - r^h\})$, and $\lambda(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}) = \sum_{h=1}^{3} \alpha^h(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r})$. Obviously, $\lambda(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}) \geq 1$ on S^3, hence the function β is continuous. We call $\boldsymbol{p}$ the 'market price' and $\boldsymbol{r}$ the 'producer price'.

The market price is determined through the standard market correspondence $\rho: \prod_i X_i \times S \rightarrow S$ defined by

$$\rho(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n, \boldsymbol{q}) = \left\{\boldsymbol{p} \in S \middle| \boldsymbol{p}\left(\sum_i \boldsymbol{x}_i - \sum_i \omega_i - g(\boldsymbol{q})\right) \geq \boldsymbol{p}'\left(\sum_i \boldsymbol{x}_i - \sum_i \omega_i - g(\boldsymbol{q})\right) \text{ for all } \boldsymbol{p}' \in S\right\} \tag{18}$$

The continuity of g ensures that this correspondence is upper hemi-continuous, with non-empty and convex values.

The price of the firm are determined through the function $\psi: S \rightarrow S$ defined by

$$\psi(\boldsymbol{p}, \boldsymbol{q}) = \hat{\psi}(g(\boldsymbol{q}) \,|\, g(\boldsymbol{q}), \boldsymbol{p}), \tag{19}$$

where $\hat{\psi}$ is the pricing rule defined previously. The correspondence ψ is upper hemi-continuous and compact and convex valued on S^2.

The fixed point mapping $\Phi: S^3 \times \prod_i \hat{X}_i \rightarrow S^3 \times \prod_i \hat{X}_i$ is defined as

$$\Phi(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) = \rho(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n, \boldsymbol{q}) \times \beta(\boldsymbol{p}, \boldsymbol{r}, \boldsymbol{q}) \times \psi(\boldsymbol{q}) \times \prod_{\boldsymbol{i}} \xi_{\boldsymbol{i}}(\boldsymbol{p}, \boldsymbol{q}). \tag{20}$$

By the Kakunani's fixed point theorem, Φ has a fixed point $(\hat{\boldsymbol{p}}, \hat{\boldsymbol{q}}, \hat{\boldsymbol{r}}, \hat{\boldsymbol{x}}_1, \ldots, \hat{\boldsymbol{x}}_n)$, and we set $\hat{\boldsymbol{y}} = g(\hat{\boldsymbol{q}})$ and $\hat{\boldsymbol{z}} = \sum_i \hat{\boldsymbol{x}}_{\boldsymbol{i}} - \hat{\boldsymbol{y}} - \sum_i \omega_i$. Then $\hat{\boldsymbol{y}} \in \partial Y$ and the following conditions hold;

$$\hat{\boldsymbol{q}} = \beta(\hat{\boldsymbol{p}}, \hat{\boldsymbol{q}}, \hat{\boldsymbol{r}}), \tag{21}$$

$$\hat{\boldsymbol{x}}_i \in \xi_{\boldsymbol{i}}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{q}}), \tag{22}$$

$$\boldsymbol{p}\hat{\boldsymbol{z}} \leq \hat{\boldsymbol{p}}\hat{\boldsymbol{z}} \text{ for all } \boldsymbol{p} \in S, \tag{23}$$

$$\hat{\boldsymbol{r}} \in \psi(\hat{\boldsymbol{q}}). \tag{24}$$

Now define $\hat{\lambda} = \lambda(\hat{\boldsymbol{p}}, \hat{\boldsymbol{q}}, \hat{\boldsymbol{r}})$. Then (25) implies

$$\hat{\lambda}\hat{q}^h \geq \hat{q}^h + \hat{p}^h - \hat{r}^h, \ \ h = 1, 2, 3, \tag{25}$$

In (28), the equality holds whenever $\hat{q}^h > 0$. Multiplying both sides of (25) by $\hat{q}^h$ and summing over h, we get $(\hat{\lambda} - 1)\hat{\boldsymbol{q}}\hat{\boldsymbol{q}} = (\hat{\boldsymbol{p}} - \hat{\boldsymbol{r}})\hat{\boldsymbol{q}} \geq 0$, where $\hat{\lambda} \geq 1$ and $\hat{\boldsymbol{q}}\hat{\boldsymbol{q}} > 0$. We therefore have

$$(\hat{\boldsymbol{p}} - \hat{\boldsymbol{r}})\hat{\boldsymbol{q}} \geq 0. \tag{26}$$

By the definition of the map ν, there exists $\hat{\mu} > 0$ such that $\hat{\boldsymbol{q}} = \hat{\mu}(\hat{\boldsymbol{y}} + k\boldsymbol{1})$. Using the fact that $(\hat{\boldsymbol{p}} - \hat{\boldsymbol{r}})\boldsymbol{1} = 0$, we get $(\hat{\boldsymbol{p}} - \hat{\boldsymbol{r}})\hat{\boldsymbol{q}} = \hat{\mu}(\hat{\boldsymbol{p}} - \hat{\boldsymbol{r}})\hat{\boldsymbol{y}}$, which combined with (26) gives

$$\hat{\boldsymbol{p}}\hat{\boldsymbol{y}} \geq \hat{\boldsymbol{r}}\hat{\boldsymbol{y}}. \tag{27}$$

By (24), $\hat{\boldsymbol{r}}\hat{\boldsymbol{y}} \geq 0$ and therefore $\hat{\boldsymbol{p}}\hat{\boldsymbol{y}} \geq 0$, hence it follows from (C–5) that $\hat{\boldsymbol{p}}\omega_i + \theta_i \hat{\boldsymbol{p}}\hat{\boldsymbol{y}} > \inf \hat{\boldsymbol{p}}X_i$. Combined with (26), we obtain the budget constraint for each i. Summing over all budget constraints, one has $\hat{\boldsymbol{p}}\hat{\boldsymbol{z}} \leq 0$ which, combined with (23), gives $\hat{\boldsymbol{z}} \leq 0$. The fixed point therefore defines an attainable state and consequently $\nu^{-1}(\hat{\boldsymbol{q}}) \gg 0$. Hence by (15), we have $\hat{\boldsymbol{q}} \gg 0$, and by (25), $\hat{\lambda} = 1$ and $\hat{\boldsymbol{p}} = \hat{\boldsymbol{r}}$. The condition (E–1) follows from (22) by a standard argument (See Debreu (1959)). Hence $\hat{\boldsymbol{p}} \gg 0$ by the assumption (C–4). Combined with $\hat{\boldsymbol{p}} = \hat{\boldsymbol{r}}$, the conditions (E–2) and

(E-3) are established by (26). One the other hand, the assumption (C-4) implies local nonmatiation. As a consequence, the budget constraints hold with equality, hence the condition (E-4) follows from $\hat{\boldsymbol{p}}\hat{\boldsymbol{z}} = 0$ and(23). Then we have established that $(\hat{\boldsymbol{p}}, (\hat{\boldsymbol{x}}_i), \hat{\boldsymbol{y}})$ is an equilibrium. ▮

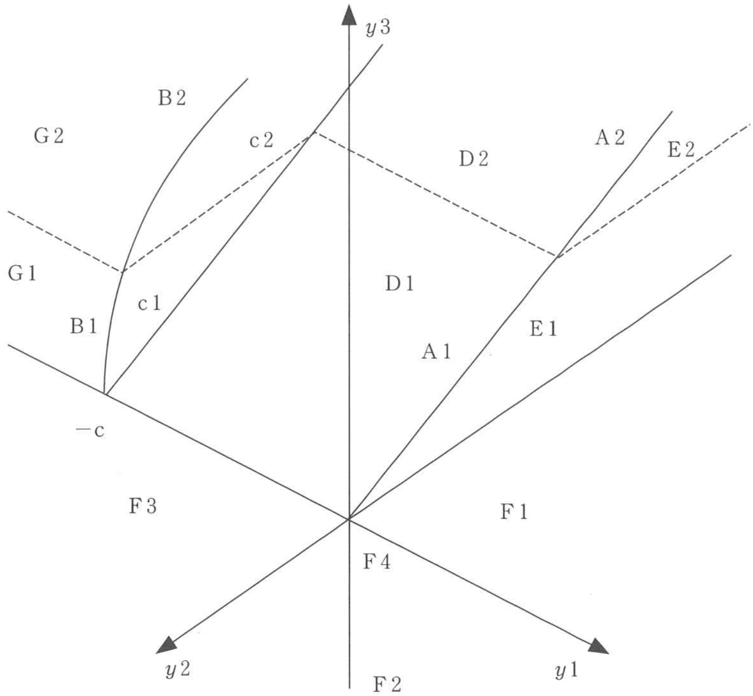

Figure 1

References

Arrow, K. J, and F. Hahn, 1971, *General Competitive Analysis,* North-Holland, Netherlands.

Arrow, K. J, and G. Debreu, 1954, "Existence of an equilibrium for a competitive economy", *Econometrica* 22, 265–290.

Brown, D. J., G. M. Heal, M. Ali Khan, and R. Vohra, 1986, "On a General Existence Theorem for Marginal Cost Pricing Equilibria", *Journal of Economic Theory* 38, 371–379.

Chamberlin, E. H., 1956. *The Theory of Monopolistic Competition,* 7th ed. Harvard University Press, Cambridge.

Debreu, G., 1959, *Theory of Value,* Wiley, New York.

Debreu, G., 1962, "New Concepts and Techniques for Equilibrium Analysis", *International Economic Review* 3, 257–273.

Dehez, P., and J. Dreze, 1988, "Competitive Equilibria with Quantity-Taking Producers and Increasing Returns to Scale", *Journal of Mathematical Economics* 17, 209–230.

McKenzie, L., 1959, "On the existence of general equilibrium for a competitive market", *Econometrica* 27, 54–71.

Negishi, T., 1961, "Monopolistic Competition and General Equilibrium", *Review of Economic Studies* 28, 196–201.

Nikaido, H., 1956, "On the classical multilateral exchange problem", *Metroeconomica* 8, 135–145.

Robinson, J., 1933, *The economics of imperfect competition,* Macmillan, London.

(2004年5月7日経済学会受理)