『経済研究』（明治学院大学）第 140・141 合併号　2008 年

Regular Economies with Infinitely Many Commodities

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Ⅰ. Introduction

In a seminar paper, Debreu [1970] started the study of the local uniqueness (finiteness) of competitive equilibria of exchange economies. He considered a topological space of all (smooth) economies and concluded that the economies with locally finite and structurally stable equilibria are open and dense subset of the space. Such economies are now called the regular economies. This result has been extended to several directions, for example, to production economies, the exchange economies with a continuum of traders and so on by Smale [1974], Dieker [1975], Mas-Colell [1975, 1977], Kehoe [1983], Rajan [1977] and many others. The reader can consult the book of Mas-Colell [1985] for related works and literatures.

On the other hand, Kehoe and Levine [1985], Balasko [1997] and Chichilnisky and Zhou [1998] and others have extended Debreu's result to economies with infinite dimensional commodity spaces. This note is a modest contribution to this direction of research.

When the commodity space is infinite dimensional, the study of the local uniqueness gives rise to a mathematically subtle problem. Debreu's original approach is to invoke excess demand functions and apply the differential calculus technique (the Sard's theorem) to it. In the case of infinite dimensional commodity spaces, however, the story does not proceed so smoothly as in the finite dimensional cases. First of all, Araujo [1985] observed that in general Banach spaces of commodities, the continuous demand functions exist only when the space is reflexive. Hence for ℓ^p cases, the case $p=\infty$ is excluded. Moreover, in such spaces the smooth demand functions exist only when the dual spaces coincide with the spaces themselves. Therefore in ℓ^p, only in the case $p=2$, differential calculus via excess demand functions is possible. In order to rescue this difficulty, people resort to the Negishi approach to equilibrium analysis, characterizing competitive equilibrium using a social welfare function rather than the excess domain functions. For instance, Balasko [1997] studied regular economies on the space ℓ^∞, the set of bounded sequences with supremum norm is a natural candidate for the commodity space (See Bewley [1972]). He parameterized "the equilibrium manifold" by weights of a social welfare function (the weighted sum of individual utility functions) on Pareto optimal allocations, rather than (as usual) by the price vectors through the excess demand equations. This clever trick made him to avoid the fundamental difficulty arising in the infinite dimensional settings mentioned above. Chichilnisky and Zhou [1998] dealt with $C([0,T])$ as the commodity space. It is the set of continuous functions on $[0,T]$ (continuous time interval). Their strategy to handle the problem is that they discarded price vectors which do not support equilibria and worked on a smaller subspace rather than the positive cone itself.

But this is not the end of the story. As Shannon [1999] pointed out, in the infinite dimensional commodity spaces, the boundary conditions on the indifference surface ensuring that equilibrium allocations sit within the interior of consumption sets are not so trivial compared to the finite dimensional cases. The condition that indifference surface does not intersect the boundary of the consumption set does not work. We have to handle carefully the tail of allocation vectors. This problem is relatively not so serious in both of the above cases where the topology is defined by the sup norm of the vectors. In such spaces, for instance, the vectors bounded away from 0 can be included as allocation vectors and we can restrict our attention to such well behaved consumption bundles.

In this note we will work on the Hilbert space ℓ^2, the set of square summable sequences. The ℓ^2 space has been found to be of great importance of applications such as economic analysis of financial markets. We follow the strategy of Balasko to use the Negishi approach and avoid the problem of the smoothness of excess demand functions. In order to handle the problem of boundary condition of the indifference surface, we have to find appropriate boundary conditions fitted to the topology of this space and they have not been known so far. Furthermore, the proof of showing smoothness of a technical key concept (the saving function or Negishi function) is not trivial at all. The next section presents the model and results. In appendix, we collect the mathematical concepts and related results used in our analysis.

Ⅱ. The model and results

Consider an exchange economy with m consumers. The commodity space of our economy is the Hilbert space

$$H=\ell^2=\left\{x=(x^t)\,\middle|\,\sum_{t=0}^{\infty}|x^t|^2<\infty\right\}. \tag{1}$$

It is well known that H is a separable Banach space, therefore it is a Lindelöf space.

The consumer i's consumption set X_i is defined by

$$X_i = \{\boldsymbol{x} = (x)^t | x^t > a_i,\ t = 0,1,\ldots\},\ a_i < 0, \tag{2}$$

namely that he/she can supply the commodity up to $-a_i$ unit at every period. The economic interpretation of this consumption set is natural enough, and seems to be even more appropriate than the case $a_i = 0$ where the consumption set is positive orthant in the case of production economies, although we focus on the exchange economies in the paper. The mathematical advantage of this consumption set for our analysis is that it is a convex and open subset of ℓ^2, which is much suitable for differential calculus on it. Indeed, let $\boldsymbol{x} = (x_i^t) \in X_i$. Since $\boldsymbol{x} \in \ell^2$, there exists $r > 0$ such that $|x^t| < 1/2$ for $t > r$. Let $x^q + 1 = min\{x^t + a_i | 0 \leq t \leq r\}$, and set $\epsilon = min\{x^q + a_i, 1/2\}$. Then it is easy to verify that $\boldsymbol{y} \in X_i$ for all $\boldsymbol{y}$ such that $||\boldsymbol{x} - \boldsymbol{y}|| < \epsilon$. Hence X_i is open subset of ℓ^2 in the norm topology. The convexity is obvious.

The consumer i's initial endowment vector is denoted by $\omega_i = (\omega_i^0, \omega_i^1, \ldots) \in X_i$. The total resource $\rho \in \ell^2_{++} = \{\boldsymbol{x} \in \ell^2 | x^t > 0,\ t - 0,1\ldots\}$ is assumed to be fixed. Let $\Omega = \{(\omega_1 \ldots \omega_m) \in \prod_{i=1}^m X_i | \sum_i \omega_i = \rho\}$ be the set of initial endowment vectors satisfying the resource constraint. As will be seen, we identify each vector $(\omega_1 \ldots \omega_m) \in \Omega$ as an economy.

The consumer i's preference is represented by a utility function

$$u_i : X_i \to \mathbb{R},\ \boldsymbol{x} \mapsto u_i(\boldsymbol{x}),\ i = 1 \ldots m. \tag{3}$$

We assume that the utility function satisfies the following conditions.

(U-1) u_i is twice continuously (Fréchet) differentiable, namely that of class C^2 on X_i.

Let $Du_i(\boldsymbol{x}) = (\partial_0 u_i(\boldsymbol{x}), \partial_1 u_i(\boldsymbol{x}), \ldots)$ be the derivative (tangent map) of u_i at $\boldsymbol{x} \in X_i$, and

$$D^2 u_i(\boldsymbol{x}) = \begin{bmatrix} \partial_0 \partial_0 u_i(\boldsymbol{x}) & \partial_0 \partial_1 u_i(\boldsymbol{x}) & \cdots \\ \partial_1 \partial_0 u_i(\boldsymbol{x}) & \partial_1 \partial_1 u_i(\boldsymbol{x}) & \cdots \\ \cdots\cdots\cdots & \cdots\cdots\cdots & \cdots \end{bmatrix} \tag{4}$$

be the second derivative, where $\partial_t u_i(\boldsymbol{x}) = \lim_{h \to 0} (1/h)(u_i(x^0, \ldots, x^t + h, \ldots) - u_i(x^0, \ldots, x^t, \ldots))$, and so on. For every $\boldsymbol{x} \in X_i$, $D^2 u_i(\boldsymbol{x})$ is considered to be a linear map from ℓ^2 to itself. We assume for every i,

(U-2) u_i is strictly differentiably monotone, i.e., $Du_i(\boldsymbol{x}) \gg 0$ for every $\boldsymbol{x} \in X_i$.

(U-3) $D^2 u_i(\boldsymbol{x})$ is a nondegenerate, negative definite bilinear form on ℓ^2,

i.e., $(\boldsymbol{y}, D^2 u_i(\boldsymbol{x}) \boldsymbol{y}) \leq 0$ for every $\boldsymbol{y} \in \ell^2$ and the equality holds only when $\boldsymbol{y} = 0$.

(U-4) for every sequence $\boldsymbol{x}_n = (x_n^t) \in X_i$ such that $x_n^t \to a_i$, for some t, it follows that $u_i(\boldsymbol{x}) \to -\infty$.

Note that the assumption (U-3) implies that the linear map $D^2 u_i(\boldsymbol{x})$ is a linear isomorphism of X_i to itself. The assumption (U-4) will be sometimes called Inada condition.

The list $\{u_i,\omega_i\}_{i=1}^{m}$ is called an economy and denoted by ε. From now on, we fix the utility functions and parameterize economy by the endowment vectors. Therefore the set $\Omega=\{(\omega_1\ldots\omega_m)\in\prod_i\Omega_i|\sum_i\omega_i=\rho\}$ is the space of all economies.

A price vector in the economies is a sequence $\boldsymbol{p}=(p^t)\gg 0$ with $\sum_{t=0}^{\infty}|p^t|^2<+\infty$, an element of ℓ^2_{++}. For a given commodity vector $\boldsymbol{x}=(x^t)$, the value of $\boldsymbol{x}$ evaluated by the price $\boldsymbol{p}$ is denoted by $\boldsymbol{px}=\sum_{t=0}^{\infty}p^t x^t$. An m-tuple of consumption vectors $(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_m)\in\prod_i X_i$ is called an allocation. The allocation is said to be feasible if $\sum_i\boldsymbol{x}_i=\sum_i\omega_i(=\rho)$.

Definition 1: An allocation $(\boldsymbol{x}_i)$ is said to be Pareto optimal if it is a solution of the following social welfare maximization problem;

$$\text{Given } \lambda=(\lambda_2\ldots\lambda_m)\in\mathbb{R}^{m-1}_{++},\text{ maximize } u_1(\boldsymbol{x}_1)+\textstyle\sum_{i=2}^{m}\lambda_i u_i(\boldsymbol{x}_i)\ \text{ subject to } \sum_i\boldsymbol{x}=\rho.$$

We denote the solution associated with λ by $\boldsymbol{x}_i(\lambda)$. Obviously, $\boldsymbol{x}_i(\lambda)=(x_i^t(\lambda))\in X_i$. Indeed, since $\boldsymbol{x}(\lambda)\in\ell^2$, we have $x_i^t(\lambda)\to 0\ (t\to\infty)$. Hence $x_i^t(\lambda)>a_i$ for all t large enough. By the Inada condition (U-4), it is impossible that $x_i^t(\lambda)=a_i$ for any t.

The Negishi equation (Balasko [1997a]) then express the property that the equilibrium allocation of a given economy are the Pareto optimal that satisfy the budget constraints of consumers, namely that

$$\nu_i(\lambda,\omega)=Du_i(\boldsymbol{x}_i(\lambda))(\boldsymbol{x}_i(\lambda)-\omega_i)=0,\ i=2\ldots m. \tag{5}$$

Of course the first consumer's budget equation follows from the Walrus law; $\sum_{i=1}^{m}Du_i(\boldsymbol{x}_i)(\boldsymbol{x}_i-\omega_i)=0$ and the first order conditions of the maximization problem, namely that $Du_1(\boldsymbol{x}_i)=\lambda_i Du_i(\boldsymbol{x}_i)$, $i=2\ldots m$. The next definition is due to Balasko [1997a].

Definition 2: A pair $(\lambda,\omega)\in\mathbb{R}^{m-1}_{++}\times\Omega$ is an equilibrium if it is a solution of the Negishi equation systems (5).

Example: There exists one consumer. The consumption set is given by

$$X=\{\boldsymbol{x}=(x^t)\in\ell^2|x^t>-1,\ t=0,1,\ldots\}, \tag{6}$$

and the utility function is of time separable form,

$$u(\boldsymbol{x})=\sum_{t=0}^{\infty}\delta^t\log(x^t+1),\ 0<\delta^t<1. \tag{7}$$

Then we have,

$$Du(\boldsymbol{x})=(\frac{1}{x^0+1},\frac{\delta}{x^1+1},\ldots), \tag{8}$$

and

$$D^2u(\boldsymbol{x}) = -\begin{bmatrix} \frac{1}{(x^0+1)^2} & 0 & 0 & \cdots \\ 0 & \frac{\delta}{(x^1+1)^2} & 0 & \cdots \\ \cdots\cdots & \cdots\cdots & \cdots & \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots \end{bmatrix}. \tag{9}$$

Let $G \subset \mathbb{R}^{m-1}_{++} \times \Omega$ denote the set of equilibria. The natural projection $\pi : G \to \Omega$ is the restriction to G of the projection $(\lambda, \omega) \to \omega$.

Theorem 1: The Negishi function $\nu = (\nu_2 \ldots \nu_m)$ is smooth.

Proof: First we shall show that the map $\lambda \to x_i(\lambda)$, $i = 2 \ldots m$ is smooth. Note that the function is determined implicitly by the first order condition of the social welfare maximization problem (5) with a resource constraint;

$$\sum_i x_i - \rho = 0,\ Du_1(\boldsymbol{x}_1) - \lambda_i Du_i(\boldsymbol{x}_i) = 0,\ i = 2 \ldots m. \tag{10}$$

It is sufficient to check that the Jacobian matrix of the above equation system

$$\begin{bmatrix} I & I & I & \cdots & I \\ -D^2u_1 & \lambda_2 D^2u_2 & 0 & \cdots & 0 \\ -D^2u_1 & 0 & \lambda_3 D^2u_3 & \cdots & 0 \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots & \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots \\ -D^2u_1 & 0 & 0 & \cdots & \lambda_m D^2u_m \end{bmatrix} \tag{11}$$

is invertible (Theorem A1). This matrix is equivalent to

$$\begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ -D^2u_1 & D^2u_1 + \lambda_2 D^2u_2 & D^2u_1 & \cdots & D^2u_1 \\ -D^2u_1 & D^2u_1 & D^2u_1 + \lambda_3 D^2u_3 & \cdots & D^2u_1 \\ \cdots\cdots & \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots & \cdots & \\ \cdots\cdots & \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots & \cdots & \cdots\cdots\cdots\cdots \\ -D^2u_1 & D^2u_1 & D^2u_1 & \cdots & D^2u_1 + \lambda_m D^2u_m \end{bmatrix}. \tag{12}$$

Therefore it suffices to show that the matrix

$$M=\begin{bmatrix} D^2u_1+\lambda_2D^2u_2 & D^2u_1 & \cdots & D^2u_1 \\ D^2u_1 & D^2u_1+\lambda_3D^2u_3 & \cdots & D^2u_1 \\ D^2u_1 & \cdots\cdots\cdots & \cdots & \\ \cdots\cdots\cdots & \cdots\cdots\cdots & \cdots & \cdots\cdots\cdots \\ D^2u_1 & D^2u_1 & \cdots & D^2u_1+\lambda_mD^2u_m \end{bmatrix} \tag{13}$$

is invertible. Let $\upsilon=(\upsilon_2\cdots\upsilon_m)\in(\ell^2)^{m-1}$ and consider the equation $Mv=0$. Then we have

$$(D^2u_1+\lambda_jD^2u_j)v_j+D^2u_1\Big(\sum_{k\neq j}v_k\Big)=0,\ j=2\ldots m. \tag{14}$$

Making the inner product with the vector v_j then yields

$$\Big(v_j,(D^2u_1+\lambda_jD^2u_j)v_j\Big)+\Big(v_j,D^2u_1\Big(\sum_{k\neq j}v_k\Big)\Big)=0,\ j=2\ldots m, \tag{15}$$

which simplifies into

$$(\lambda_jv_j,D^2u_jv_j)+\Big(v_j,D^2u_1\Big(\sum_{k\neq 2}^{m}v_k\Big)\Big)=0,\ j=2\ldots m. \tag{16}$$

We add up these equations for i from 2 to m,

$$\sum_{j\neq 2}^{m}\lambda_j(v_j,D^2u_jv_j)+\Big(\Big(\sum_{j=2}^{m}v_j\Big),\ D^2u_1\Big(\sum_{j=2}^{m}v_j\Big)\Big)=0. \tag{17}$$

Since D^2u_j are negative definite, each term in the sum of the left hand side of (14) is ≤ 0, hence each term must be 0,

$$\Big(\Big(\sum_{j=2}^{m}v_j\Big),\ D^2u_1\Big(\sum_{j=2}^{m}v_j\Big)\Big)=0,\ \lambda_j(v_j,D^2u_jv_j)=0,\ j=2\ldots m. \tag{18}$$

Therefore we have $v_2=\cdots=v_m=0$. This shows that the orthonormal basis $\{(1,0,0,\ldots),(0,1,0,\ldots),\ldots\}$ maps to an independent set, which can be transformed to an orthonormal basis by the Gram-Schmidt procedure. It is well known that the two Hilbert spaces with the bases of the same cardinality are isomorphic, hence the matrix M is invertible. The smoothness of ν_i follows from the smoothness of Du_i and the inner product.

Theorem 2: The set of equilibria G is a smooth submanifold of $\mathbb{R}^{m-1}\times\Omega$ of codimension $m-1$.

Proof: We shall prove this proposition by applying the infinite dimensional version of the regular value theorem (Theorem A2, appendix), which amounts to prove that the map ν, which is smooth by Proposition 1, is transversal to the closed submanifold $\{0\}\subset\mathbb{R}^{m-1}$. Let $D\nu(\lambda,\omega)$ denote the tangent map (derivative) of ν at $(\lambda,\nu)\in G$. In order to show the transversality, we have to verify (i) the map $D\nu(\lambda.\omega):\mathbb{R}^{m-1}\times T_\omega\Omega\to\mathbb{R}^{m-1}$ is onto, and (ii) the kernel $N=D\nu(\lambda,\omega)^{-1}(0)$ has a closed complement in $\mathbb{R}^{m-1}\times T_\omega\Omega$, i.e., there exists a closed subspace H such that $N\oplus H=\mathbb{R}^{m-1}\times T_\omega\Omega$. First we consider the topological complement of the tangent map, which is defined by the $m-1$

coordinate mappings of the derivatives of ν_i with respect to $(\lambda,\omega_1,\dots,\omega_m)$. Note that the partial derivative of ν_i with respect to $\omega_i(j\neq i)$ is 0. The derivative of ν_i is then written as

$$Dv_i(\lambda_j,\omega_i)(\mu,\xi)=\frac{\partial\nu_i(\lambda,\omega_i)}{\partial\lambda}\mu+\sum_{t=0}^{\infty}\frac{\partial\nu_i(\lambda,\omega_i)}{\partial\omega_i^t}\xi^t \tag{19}$$

for $\mu\in\mathbb{R}^{m-1}$ and $\xi=(\xi^t)\in\ell^2$. The linear functional $D\nu_i(\lambda,\omega_i)$ is continuous for the norm topology, so its kernel is a closed hyperplane in the Banach space $\mathbb{R}^{m-1}\times T_\omega\Omega$. The kernel of the tangent map $D\nu=(D\nu_1,\dots,D\nu_m)$ is therefore the closed subspace of finite codimension defined by the intersection of $m-1$ closed hyperplanes. Hence it has a topological complement.

We now consider the surjectivity of the tangent map. It suffices that we prove the surjectivity of the restriction of this map to some subspace of $\mathbb{R}^{m-1}\times T_\omega\Omega$. Pick an arbitrary period t. The Jacobian matrix of $D\nu$ with respect to $(\omega_2^t\dots\omega_m^t)$ is

$$\begin{bmatrix} \partial_t u_2(\boldsymbol{x}_2(\lambda)) & 0 & \cdots & 0 \\ 0 & \partial_t u_3(\boldsymbol{x}_3(\lambda)) & \cdots & 0 \\ \cdots\cdots & \cdots\cdots & \cdots & \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots \\ 0 & 0 & \cdots & \partial_t u_m(\boldsymbol{x}_m(\lambda)) \end{bmatrix}, \tag{20}$$

which is obviously of rank $m-1$ by the assumption (U-2).

By Proposition 2, it follows that the projection map $\pi: G\to\Omega, (\lambda,\omega)\mapsto\omega$ is smooth, since it is a composition of smooth maps, the canonical embedding $G\to\mathrm{R}_{++}^{m-1}\times\Omega$ and the canonical projection map $\mathrm{R}_{++}^{m-1}\times\Omega\to\Omega$.

Definition 3: An economy $\omega\in\Omega$ is said to be regular if it is a regular value of the projection $\pi: G\to\Omega, (\lambda,\omega)\mapsto\omega$.

Our main result reads as follows.

Theorem 3: The set of regular economies is a dense G_δ subset of Ω.

Proof: It follows from theorem A3 in Appendix that the natural projection $\pi: G\to\Omega$ is Fredholm map of index 0, since the codimension of G is equal to $m-1$, hence by theorem A4, the set of regular economies, which is the set of regular values of π is the residual subset of Ω.

Ⅲ. Appendix

In this appendix, we collect mathematical concepts and results which are used in the text for reader's convenience.

Let E and F be Banach spaces and $U \subset E$ be open. A map $f: U \to F$ is (Fréchet) differentiable at $x \in U$ if there is a continuous linear map $Df(x): E \to F$ such that

$$\lim_{h \to \infty} \frac{||f(x+h) - f(x) - Df(x)||}{||h||} = 0 \tag{21}$$

Let $L(E,F)$ denote the Banach space of continuous linear map from E to F under the operator norm, namely that for $T \in L(E,F)$,

$$||T|| = sup\left\{||Tx|| \middle| ||x|| \leq 1\right\}. \tag{22}$$

For every integer $k \geq 0$, the map f is said to be of class C^k at $x \in U$ if the map

$$D^k f(x) \equiv D(D^{k-1}f)(x): U \to L(E, L^{k-1}(E,F)) \equiv L^k(E,F) \tag{23}$$

is continuous. Let $E = E_1 \oplus E_2$ (direct sum), where E_1 and E_2 are Banach spaces, and $U \subset E$ is open. For a map of class $C^K F: U \to F$, the partial derivative with respect to E_1 is defined by

$$D_1 f(x) e_1 \equiv Df(x)(e_1, 0),\ e_1 \in E_1. \tag{24}$$

The partial derivative with respect to E_2 is defined similarly. The next theorem is of central importance in the differential calculus on manifolds (Abraham et al [1981, p.121]).

Theorem A1 (Implicit function theorem): Let $U_1 \subset E_1$, $U_2 \subset E_2$ be open and $f: U_1 \times U_2 \to F$ be of class $C^k (k \geq 1)$. For some $x_0 \in U_1$ and $y_0 \in U_2$, assume that $D_2 f(x_0, y_0): E_2 \to F$ is an isomorphism. Then there are neighborhoods U_0 of x_0 and $V \subset F$ of $f(x_0, y_0)$, and a unique map $g: U_0 \times V \to U_2$ such that for all $(x,z) \in U_0 \times V$,

$$f(x, g(x,z)) = z.$$

A manifold M modeled by a Banach space E is defined similarly as in the case of $E = \mathbb{R}^n$; A topological space M is a Banach manifold if there exist an open cover $\{U_\alpha\}$ and local isomorphisms ϕ_α on U_α to E such that $\phi_\beta \circ \phi_\alpha^{-1}: \phi_\alpha(U_\alpha \cap U_\beta) \to \phi_\beta(U_\alpha \cap U_\beta)$ is of class C^k for each α and β. Let $p \in M$. A C^k curve through p is a C^k map from $(-\epsilon, \epsilon)$ to M such that $c(0) = p$. Two curves c and d are equivalent if and only if $\dot{c}(0) (\equiv dc(0)/dt) = \dot{d}(0)$. The equivalence class is denoted as $[c]$ and called a tangent vector at p. Let $T_p M$ be the set of all tangent vectors at p and call the tangent space at p. It is easy to see that T_p is a vector space. A map f from a Banach manifold M to a Banach manifold N is said to be of class C^k if the map $\phi_\beta \circ f \circ \phi_\alpha^{-1}$ is of class C^k. The tangent map (derivative) at $p \in M$ of a C^k map f is a linear map $Df(p): T_q \to T_q N$ defined by $Df(p)([c]_p) = [f \circ c]_{f(p)}$.

Let F be a closed subspace of a Banach space E. F splits in E if there exists a closed subspace $G \subset E$ such that $E = F \oplus G$. Let $f: M \to N$ be a smooth map between two Banach manifolds. A point $q \in N$

is a regular value of f (or f is transversal to $\{q\}$) if for every $p \in f^{-1}(q)$, $Df(p)$, is surjective and the kernel $D^{-1}f(p)(\{0\})$ splits in T_pM.

Theorem A2 (Abraham et al [1981, p. 202]): Let $f: M \to N$ be a smooth map and $q \in N$ a regular value of f. Then $f^{-1}(q)$ is a submanifold of N. If N is finite dimensional, then $dimf^{-1}(q)$ (=dimension of $f^{-1}(q)$) = $dim\ N$.

A linear continuous map $\lambda: E \to F$ is called a Fredholm operator if $Ker(\lambda) \equiv \lambda^{-1}(0)$ is a finite dimensional subspace of E,and $Im(\lambda) \equiv \lambda(E)$ has a finite codimension in F. The index of the Fredholm operator, $ind(\lambda)$ is defined by

$$ind(\lambda) = dimKer(\lambda) - codimIm(\lambda) \qquad (25)$$

A C^k map f is called a Fredholm map if $Df(p)$ is the Fredholm operator at every p. We have (Abraham and Robbing [1967, pp. 45-46]),

Theorem A3: Let E and F be a Banach spaces with $dimE = n$, and assume that $\pi: E \times F \to F$ is the projection operator. If G is a closed subspace of $E \times F$ of codimension q, then the restriction $\pi: G \to F$ is a Fredholm operator of index $n - q$.

The next result which is an infinite dimensional analogue of Sard's theorem is due to Smale (Abraham and Robbing [1967, pp. 42-43]).

Theorem A4: Let $f: E \to F$ be a C^k Fredholm map between C^k Banach manifolds where E is a Lindelöf space. If at every $p \in E$, $k > max\{0, ind(Df(p))\}$, then the set of regular values of f is a residual subset of F.

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（2007 年 9 月 26 日　産業経済研究所受理）