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Existence and Optimality of Competitive Equilibria with Marshallian Increasing Returns

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1 Introduction

In this paper we will be concerned with the idea of A. Marshall on the external economies and increasing returns. The economies of the modern advanced countries are characterized by the enormous extension of the markets, the rapid development of the technologies and the further monopolization of the markets by small numbers of big companies. These characters of the markets are mutually related and, at least the last two of them belong to the Marshallian tradition of the economic thought. The modern general equilibrium theory which has essentially grown out of the Walrasian tradition so far therefore is not sufficient for considering the problems arising from the markets with these characters. In order to tackle the problems successfully, I believe that one has to go back to Marshall and develop the theory which unifies the ideas of the two great masters of the equilibrium analysis. The following sections of the present paper try to be a first step toward such a theory. I want to show in this book that the main body of the Marshallian concepts including the externalities and the increasing returns does indeed fit into the scheme of the modern general equilibrium theory.

In his "*Principles*", Marshall divided increasing returns to two types on account of their sources. His definitions are as follows.

> "We may divide the economies arising from an increase in the scale of production of any kind of goods, into two classes - firstly, those dependent on the general development of the industry; and secondly, those dependent on the resources of the individual houses of business engaged in it, on their organization and the efficiency of their management. We may call the former *external economies*, and the latter *internal economies* (1920, Chapter IX, p. 221)."

Of course, as a theorist of the 19-th century, he agreed that the limitations of production factors such as land naturally led to the decreasing returns. Whatever external or internal, he considered that increasing returns are something which are related to the efficiency of the human skill and technology.

> "... We say broadly that while the part which nature plays in production shows a tendency to diminishing return, the part which man plays shows a tendency to increasing return. The *law of increasing return* may be worded thus: - an increase of labour and capital leads generally to improved organization, which increases the efficiency of the work of labour and capital (1920, Chapter XIII, p. 265)."

Therefore it seems fair to assume that (at least) most parts of "internal economies" in Marshall's sense correspond to nonconvex production sets in the modern terminology, and "external economies" to (positive) external effects between individual firms when one tries to express his idea of economies of scale within a framework of modern equilibrium theories.

In any case, Marshall proposed these "economies of scale" as theoretical concepts, not simply as observations of actual facts. This means that he had assumed that the increasing returns whatever they were internal or external were consistent with his system as a whole, or more specifically, he assumed that the increasing returns were compatible with perfectly competitive equilibrium.

At least, Marshall had recognized the inconsistency between the internal economies and the competitive (or price taking) behavior of individual firms. A part of the reason which supports his intuition seems that his equilibrium concept of perfect competition is something which is defined over long period of time so that free entries and exits are allowed, and distinguished from the temporary equilibrium. Invoking the famous metaphor of trees of the forest, he gave an image of his equilibrium concept in which each firm lives its own life time in the market.

> "But here we may read a lesson from the young trees of the forest as they struggle upwards through the benumbing shade of their older rivals. Many succumb on the way, and a few only survive; those few become stronger with every year, they get a larger share of light and air with every increase of their height, and at last in their turn they tower above their neighbours, and seem as though they would grow on for ever, and for ever become stronger as they grow. But they do not. One tree will last longer in full vigour and attain a greater size than another; but sooner or later age tells on them all. Though the taller ones have a better access to light and air than their rivals, they gradually lose vitality; and one after another they give place to others, which, though of less material strength, have on their side the vigour of youth(1920, Chapter XIII, p. 263)."*

*In many places of The Principles, Marshall emphasized the analogy between economics and biology. His attitude toward biology is sometimes compared to that of Walras toward mechanics.

Unfortunately, by the intuitive and verbatim nature of his exposition, the "Marshallian increasing returns (or equivalently decreasing marginal costs)" invited furious debates among theorists of the younger generations. For the controversies on the increasing returns, J.S. Chipman (1965) reports;

> "Marshall's only definition consists in the statement (*Principles*, p. 266 (p. 221 in the new edition)) that external economies are those economies of scale which are "dependent on the general development of the industry." The absence of any more elaborate formal definition in Marshall's writing is so conspicuous that it must be interpreted as deliberate; Robertson used the term "evasive" (cf. Newman (1960, p. 601)). In an earlier skeptical paper Robertson (1924, p. 26), after enumerating the usual examples (including the inevitable trade journal) sighed: "we have all at some time tried to memorize and to reproduce the formidable list." In the same year, Knight (1924, p. 597) set forth his famous objection to the concept of external economies in the words: "external economies in one business unit are internal economies in some other, within the industry." ... Knight's paper was largely a criticism of the concept of external economies - as was Robertson's 1924 paper - as used both by Pigou (1920) and by Graham (1923). Graham based his argument for protection on an analysis which took for granted the compatibility of perfect competition and increasing returns; this very assumption is what was challenged by Knight, and as long as Knight's objection stood, Graham's entire argument - whatever other defects it had, and there were several - was vitiated by having this as its premise. In his reply to Knight, Graham (1925) failed to come to grips with the main issue; and Knight (1925) in his rejoinder fairly placed the burden of proof on those who believed that competitive conditions could be reconciled with increasing returns. In saying with respect to external economies that "I have never succeeded in picturing then in my mind," Knight (1925, p. 323) was undoubtedly expressing a feeling that was widespread but suppressed, owing to the authority of Marshall and Pigou (1965, pp. 740-741)."

The consequence of the controversies is summarized by 1970 paper of Chipman:

> "(The compatibility of increasing returns with perfectly competitive equilibrium) was once a lively subject of debate. The debate appears to have petered out in the 1930's, with nobody the apparent winner. That this was the outcome seems evident from later writings of some of the participants. Thus, Sir Dennis Robertson[†] presented in 1957 on account which was substantially unaltered from his contribution to the 1930 Symposium on Increasing Returns,[‡] supporting the compatibility of increasing returns with perfect competition. On the other hand, Sir Roy Harrod in 1967 was able to state flatly, without any qualification as to

[†]Sir Dennis H. Robertson, *Lectures on Economic Principles* (London: Staples Press, 1957), vol. I, Ch.IX, pp. 114-23.
[‡]The trees of the forest", Economic Journal, vol 40 (March 1930), pp. 80-89.

whether economies were internal or external, that: "Increasing returns can, of course only occur if competition is less than perfect."[§] In the contemporary international trade literature, some authors maintain that perfect competition can prevail under conditions of increasing returns, provided the economies of scale are external to individual firms;[¶] whereas others deny the compatibility of economies of scale with perfect competition under any circumstances, and with equal confidence[‖]... (1970, pp. 347-9)".

The idea of external increasing returns of A. Marshall was made clear by J.S. Chipman (1970). He called it "parametric economies of scale", since in his formulation, each firm is supposed to take a scale parameter in its production function as given and believe that it operates under constant returns to scale, but actually the parameter is affected by the total amount of the input level of the industry as a whole, and consequently, the objective production function of the firm exhibits the increasing returns to scale. We give a simple example to illustrate the idea.

Suppose that there exist v identical firms in an industry, each of which has the same production function $y=kz$, where y is output and z is input, and the coefficient k is a parameter. The firm takes k as given when it makes the production decision, so that its subjective production function is constant returns to scale, but actually k is assumed to be determined endogenously at the level of the total input of the industry, $\sum_{j=1}^{v} z_j$, where z_j is the input level of the firm j. That is, assuming that all firms use the same amount of the input z, $k=vz$ holds in equilibrium. Then the firm's objective production function is $y=vz^2$, which exhibits the increasing returns and the resulting equilibrium concept is a competitive equilibrium with production externalities.

The idea of the parametric economies of scale originally came from Edgeworth. In the midst of the desperately confusing debates on the compatibility between competitive equilibria and the increasing returns (decreasing costs), he was looking at the truth. We quote Chipman;

"The essential idea put forward by Edgeworth (1905, pp. 66-8; Papers, III, pp. 140-1) was that marginal cost was a function of a particular firm's output, and also of aggregate industrial output; and that it might be rising with respect to the former and falling with respect to the latter. According to this conception, rising marginal cost curves for the individual

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[§] Roy F. Harrod, "Increasing Returns" in *Monopolistic Competition Theory*: Studies in Impact; Essays in Honor of Edward H. Chamberlin, ed Robert E. Kuenne (NY: John Wiley and Sons, Inc., 1967), pp. 63-76.

[¶] E.g.,James Edward Mead, *A Geometry of International Trade* (London: George Allen and Unwin Ltd., 1952) p. 33.

[‖] Cf. R.G. Lipsey, "The Theory of Customs Unions: A General Survey", *Economic Journal*, vol. 70 (Sept, 1960), 496-513 (reprinted in A.E.A., *Readings in International Economics*, Homewood, Ill: Richard D.Irwin, Inc., 1968, pp. 261-78). He states on pp. 511-12 (p. 277 of the reprinted version): "It is, of course, well known that exhausted economies of scale are incompatible with the existence of perfect competition, but it is equally well known that unexhausted economies of scale are compatible with the existence of imperfect competition as long as long-run marginal cost is declining faster [sic] than the marginal revenue (quoted by Chipman)."

firms would shift downwards with a rise in industrial output, leading to a falling supply curve for the industry...

To illustrate the case, an expansion in a certain industry may make possible a further division of labor, and give rise to new categories of technicians. The contribution of each individual firm to this process may be so negligible that no single entrepreneur will take into account the effect of his own scale of operations on the development of new specialized skills. This element of cost therefore plays the same role as do market prices. It is curious indeed that Edgeworth, of all people, did not notice the analogy between this concept of external economies and his own limit theorem justifying the competitive price mechanism (cf. Edgeworth (1881, pp. 240-3)). All we need to assume is that a firm's size has a small effect (negligible from its point of view) on the organization of the industry (especially the labor market), and that the the firm consciously adjusts its organization to the changed condition of the industry (1965, p. 740)."

At the first time in the history of this subject, Chipman (1970) showed that the external increasing returns were indeed compatible with competitive equilibrium and even Pareto optimality in one consumer economy. In the next section, we will present the concept of the external increasing returns in most general form and generalize the results of Chipman to prove the existence of the equilibrium and the Pareto optimality and the tax-subsidy policy in the presence of the increasing returns will be also discussed.

2 Production Economies with External Increasing Returns

Let L be an ordered vector space which is to be understood as the commodity space. In this section, L is the l-dimensional space $\mathbb{R}^l$. Let $L_+=\{\boldsymbol{x}\in L\,|\,\boldsymbol{x}\geq\boldsymbol{0}\}$ be the nonnegative orthant of L. Consider the function

$$F : L_+\times L_+\to L_+,\ (\boldsymbol{z}, \boldsymbol{k})\mapsto \boldsymbol{y}=F(\boldsymbol{z}, \boldsymbol{k}),$$

where $\boldsymbol{z}\in L_+$ stands for input vector, $\boldsymbol{y}\in L_+$ output and $\boldsymbol{k}\in L_+$ is a parameter. The function F is called a technology function, which is distinguished from the (standard) production function.

Definition 2.1 The technology function $F(\boldsymbol{z}, \boldsymbol{k})$ is said to exhibit external increasing returns to scale (or social increasing returns to scale) if $F(\boldsymbol{z}, \boldsymbol{k})$ is of homogeneous of degree 1 in $\boldsymbol{z}$,

$$F(\lambda\boldsymbol{z}, \boldsymbol{k})=\lambda F(\boldsymbol{z}, \boldsymbol{k}) \text{ for all } \lambda\geq 0 \text{ and all } (\boldsymbol{z}, \boldsymbol{k})\in \mathrm{L}_+\times \mathrm{L}_+,$$

and it is of monotonically increasing in $\boldsymbol{k}$,

$$\boldsymbol{k}\leq\boldsymbol{k}' \text{ implies } F(\boldsymbol{z}, \boldsymbol{k})\leq F(\boldsymbol{z}, \boldsymbol{k}') \text{ for all } \boldsymbol{z}\in L_+$$

The firms will be assumed to take the parameter $\boldsymbol{k}$ as given when it maximizes the profit, on the other hand, $\boldsymbol{k}$ will be determined endogenously at the aggregate input level, $\boldsymbol{k}=\sum\boldsymbol{z}$ in equilibrium, where the summation is over all firms in the industry, or sometimes in the economy as a whole, depending on the range of externalities. When the function F is of the external returns to scale, we see that it exhibits the "increasing returns" socially, for

$$\lambda F(\boldsymbol{z}, \boldsymbol{k})=F(\lambda \boldsymbol{z}, \boldsymbol{k}) \leq F(\lambda \boldsymbol{z}, \lambda \boldsymbol{k}) \text{ for all } \lambda \geq 1.$$

In the following, the technology function will be sometimes given by the (usual) production function with a parameter,

$$f: \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+,\ (z, k) \mapsto y=f(z, k).$$

We say that the production function f is also of the the external increasing returns if $f(z, k)$ is of homogeneous of degree 1 in z, and it is monotonically increasing in k.

Example 2.1 (Chipman (1970)) There exist l industries, each producing a single homogeneous commodity from a single factor, or the labor z^0. Every firm in the industry $t(=1...l)$ has the same production function

$$y^t= k_t^{\varepsilon t-1} z^t,\ \varepsilon_t > 0$$

which is of the external increasing returns to scale. If there exist v_t firms in the industry t, assuming that they operate under the same level of input, $k_t=v_t z^t$ will hold in quilibrium.Then the firm's objective production function is

$$y^t=k_t(z^t)^{\varepsilon t},\ k_t=(v^t)^{\varepsilon t-1},\ t=1...l.$$

Chipman assumed that there existed m consumers with the same utility function

$$u(x^1...x^l)=\sum_{t=1}^{l} \beta_t \log x^t.$$

Note that the utility function does not depend on the commodity 0, the labor. Hence each consumer supply the labor inelastically up to the amount which he/she initially holds. Assume that all consumer has the one unit of labor and zero amount of other commodities, hence the initial endowment vector of all consumers is $(1, 0, ... 0) \in \mathbb{R}^{l+1}$.

The competitive equilibrium is a triple of vectors $(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{z})=((p^t), (x^t), (z^t)) \in \mathbb{R}^l_+ \times \mathbb{R}^l_+ \times \mathbb{R}^l_+$ which satisfies that

$$\boldsymbol{px}=w \text{ and } \beta^t=\lambda p^t x^t,$$

$$\boldsymbol{py}=\boldsymbol{z}, \text{ where } \boldsymbol{y}=(y^t)=(k_t^{\varepsilon_t -1} z^t),$$

$$\sum_{t=1}^{l} v_t z^t=m,\ k_t=v_t z^t,$$

where λ is the multiplier, and from the above equations, we have $w\lambda=\sum_{t=1}^{l} \beta^t$. In the next section, we will discuss the welfare property of the competitive equilibrium for a (generalized) Chipman model.

Example 2.2 (Romer (1986)) For simplicity, we present a discrete time version of the model of Romer (1986), which is an optimal growth model of the infinite time horizon. At each time period $t+1 \geq 1$, the output commodity y^{t+1} is produced from the input commodity z^t which was available at the previous period $t \geq 0$ through the external increasing returns to scale technology,

$$y^{t+1}=f(z^t, k^t),\ t=0, 1\ldots$$

and in equilibrium, $k^t=z^t$ for all t should hold. The representative consumer's preference is given by a time separable utility function

$$U(\boldsymbol{x})=\sum_{t=0}^{\infty}\delta^t u(x^t) \text{ for } \boldsymbol{x}=(x^t),$$

where $0 \leq \delta \leq 1$ is a discount factor. At the initial period $t=0$, he/she has the initial endowment $\omega^0>0$. The competitive equilibrium is a triple of vectors $(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{z})$ which is characterized as

$$\boldsymbol{px} \leq p^0\omega^0 \text{ and } U(\boldsymbol{x}) \geq U(\boldsymbol{x}') \text{ whenever } \boldsymbol{px}' \leq p^0\omega^0,$$

$$p^{t+1} f(z^t, z^t) - p^t z^t=0,\ \mathrm{t}=0, 1, \ldots$$

$$\boldsymbol{x}+\boldsymbol{z} \leq (\omega^0, y^1, y^2\ldots),\ y^{t+1}=f(z^t, z^t),\ t=0, 1\ldots$$

In Suzuki (2008), we proved the existence of the competitive equilibrium for a (generalized) Romer model.

In the following, suppose that there exist m consumers in the economy $a=1\ldots m$. As usual, the consumer a has the preference-consumption set pair $(X_a, \prec_a)$ and the initial endowment vector $\omega_a \in \mathbb{R}^l$. The consumption set X_a is assumed to be a closed and convex subset of $\mathbb{R}^l$ which is bounded from below. We also assume that there exist n firms in the economy indexed by $b=1\ldots n$. The production technology of the firm b is described by the external increasing returns to scale technology function

$$F_b : \mathbb{R}^l_+ \times \mathbb{R}^l_+ \to \mathbb{R}^l_+,\ (\boldsymbol{z}_b, \boldsymbol{k}_b) \mapsto \boldsymbol{y}_b=F_b(\boldsymbol{z}_b, \boldsymbol{k}_b),\ b=1\ldots n.$$

Then the production set of the firm b, $Y(\boldsymbol{k}_b)$ is defined by

$$Y_b(\boldsymbol{k}_b)=\{\boldsymbol{y}_b - \boldsymbol{z}_b \in \mathbb{R}^l \mid \boldsymbol{y}_b, \boldsymbol{z}_b \geq \boldsymbol{0},\ \boldsymbol{y}_b \leq F_b(\boldsymbol{z}_b, \boldsymbol{k}_b)\},\ b=1\ldots n.$$

which depends on the parameter $\boldsymbol{k}_b \in \mathbb{R}^l_+$.

Note that for each $\boldsymbol{k}_b$, the set $Y_b(\boldsymbol{k}_b)$ is a convex cone with the vertex at $\boldsymbol{0}$, and satisfies for each $\boldsymbol{k}_b \in \mathbb{R}^l_+$,

(NP) (No production) $\boldsymbol{0} \in Y_b(\boldsymbol{k}_b)$,

and

(FD) (Free disposability) $\mathbb{R}^l_- \subset Y_b(\boldsymbol{k}_b)$.

Obviously, the correspondence

$$Y_b : \mathbb{R}^l_+ \to \mathbb{R}^l_+,\ \boldsymbol{k}_b \mapsto Y(\boldsymbol{k}_b)$$

is continuous if and only if the function $F_b : \mathbb{R}^l_+ \times \mathbb{R}^l_+ \to \mathbb{R}^l_+$ is continuous.

We call the $3m+n$-tuple $\varepsilon_k=(X_a, \prec_a, \omega_a, F_b)$ an economy with the external increasing returns or simply an economy.

The equilibrium concept for the economy $\varepsilon_k=(X_a, \prec_a, \omega_a, F_b)$ is the competitive equilibrium with the externalities.

Definition 2.2 An $m+2n+1$-tuple $(\boldsymbol{p}, (\boldsymbol{x}_a), (\boldsymbol{y}_b, \boldsymbol{z}_b))$ is said to consist of a competitive equilibrium if and only if

(E-1) $\boldsymbol{p}\boldsymbol{x}_a \leq \boldsymbol{p}\omega_a$ and $\boldsymbol{x}_a \succsim_a \boldsymbol{x}'$ whenever $\boldsymbol{p}\boldsymbol{x}' \leq \boldsymbol{p}\omega_a$, $a=1...$m,

(E-2) $\boldsymbol{p}\boldsymbol{y} \leq \boldsymbol{p}\boldsymbol{y}_b=0$ for all $\boldsymbol{y} \in Y_b\left(\sum_{c=1}^{n} \boldsymbol{z}_c\right)$, $b=1...n$,

(E-3) $\sum_{a=1}^{m} \boldsymbol{x}_a \leq \sum_{b=1}^{n}\left(F_b\left(\boldsymbol{z}_b, \sum_{c=1}^{n} \boldsymbol{z}_c\right)-\boldsymbol{z}_b\right)+\sum_{a=1}^{m} \omega_a$

Note that by the assumption of the homogeneity of F_b in the first variable, the profit of each firm in the competitive equilibrium is equal to zero. Suzuki (2008) proved the next theorem.

Theorem 2.1 Suppose that an economy $\varepsilon_k=(X_a, \prec_a, \omega_a, F_b)$satisfies the continuity, the convexity, local nonsatiation and the minimum income condition for every $a=1...m$, and for every $b=1...n$, the function

$$F_b : \mathbb{R}^l_+ \times \mathbb{R}^l_+ \to \mathbb{R}^l_+,\ (\boldsymbol{z}, \boldsymbol{k}) \mapsto \boldsymbol{y}=F_b(\boldsymbol{z}, \boldsymbol{k}),$$

is continuous. Finally we assume that the set of feasible allocations

$$F=\left\{\left((\boldsymbol{x}_a), (\boldsymbol{y}_b, \boldsymbol{z}_b)\right) \in \prod_{a=1}^{m} X_a \times \mathbb{R}^{2l}_+ \,\middle|\, \sum_{a=1}^{m} \boldsymbol{x}_a \leq \sum_{b=1}^{n}\left(F_b\left(\boldsymbol{z}_b, \sum_{c=1}^{n} \boldsymbol{z}_c\right)-\boldsymbol{z}_b\right)+\sum_{a=1}^{m} \omega_a,\ \boldsymbol{y}_b \leq F_b\left(\boldsymbol{z}_b, \sum_{c=1}^{n} \boldsymbol{z}_c\right)\right\}$$

is bounded. Then there exists a competitive equilibrium $(\boldsymbol{p}^*, \boldsymbol{x}^*_1... \boldsymbol{x}^*_m, \boldsymbol{y}^*_1... \boldsymbol{y}^*_{n1}\ \boldsymbol{z}^*_1... \boldsymbol{z}_n{}^*)$ for ε.

3 Pareto Optimality and Tax Policies

In this section, we will discuss the Pareto optimality of the competitive equilibria in the (generalized) Chipman economy with the external increasing returns. The commodity space is assumed to be $\mathbb{R}^l=\mathbb{R}^{n+1}$. Let $(x^0, x^1...x^n) \in \mathbb{R}^{n+1}$. There exists n industries indexed by $b=1...n$. The industry b produces the commodity b, and the commodity 0 is called the labor and it is used as input and not producible.

In the industry b, there exist finitely many identical firms, each of which has the increasing returns to scale production function

$$f_b : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+,\ (z, k) \mapsto y=f\,(z, k).$$

Let z_{bv} be the input level of the v-th firm in the industry b. The aggregate input level of the industry is then $z^b=\sum_v z^b_v$. Since the firms are identical, the aggregate output level can be written as $y^b=\sum_v f_b(z^b_v, z^b)=f_b(\sum_v z^b_v, z^b)=f_b(z^b, z^b)$. Hence we do not have to specify the individual firm input level z^b_v. For notational simplicity we denote $g_b(z_b)=f_b(z_b, z_b)$.

There exist m consumers, $a=1...m$. The consumer a has the utility function on the consumption set $X_a=\{0\} \times \mathbb{R}^n_+ \approx \mathbb{R}^n_+$,

$$u_a : \mathbb{R}^n_+ \to \mathbb{R},\ \boldsymbol{x} \mapsto u_a(\boldsymbol{x}),\ a=1...m.$$

Note that we assume that the consumer's utility does not depend on the production factor (labor), the commodity 0. Therefore the consumers supply the amount of the labor which they initially own inelastically. We assume that the consumer a owns initially the labor only, hence the endowment vector of the consumer is of the form $\omega_a=(\omega_a^0, 0...0)\in\mathbb{R}^{n+1}$, $\omega_a^0>0$. Further we assume that the utility function is monotone,

if $\boldsymbol{x}, \boldsymbol{z} \in X_a$ and $\boldsymbol{x}<\boldsymbol{z}$, then $u_a(\boldsymbol{x})<u_a(\boldsymbol{z})$.

Although this economy does not satisfy the minimum income condition, we can demonstrate that there exists a competitive equilibrium using the Negishi method of proof. Since it exploits the welfare theorem, it gives us a lot of information on the optimality of equilibria, and it is a basis of our discussion of tax policies implementing the social optima.

Let F be the set of feasible allocations

$$F=\left\{((\boldsymbol{x}_a), \boldsymbol{z})=((x_a^b), (z^b))\in\mathbb{R}_+^{mn}\times\mathbb{R}_+^n \;\middle|\; \sum_{a=1}^m \boldsymbol{x}_a\leq \boldsymbol{y}=(y^b),\ y^b= f_b\left(z^b, \sum_{c=1}^n z^c\right), 0\leq z^b\leq\sum_{a=1}^m\omega_a^0, b=1...n.\right\}$$

Since $0\leq\boldsymbol{x}_a\leq\sum_{a=1}^m \boldsymbol{x}_a\leq f_b\left(\sum_{c=1}^n\omega_c^0, \sum_{c=1}^n\omega_c^0\right)$, the set F is bounded. Let K be a compact and convex subset of $\mathbb{R}^{mn}\times\mathbb{R}^n$ such that $F\subset$ *interior* K.

Consider the following constrained social optimization problem $P(k)$.

$$P(k): \text{Given } k=(k_b)\in\mathbb{R}_+^n \text{ and } \alpha=(\alpha^a)\in\mathbb{R}_+^m \text{ with } \sum_{a=1}^m\alpha^a=1,$$

$$\text{maximize } \sum_{a=1}^m\alpha^a u_a(\boldsymbol{x}_a) \text{ subject to}$$

$$\sum_{a=1}^m x_a^b\leq f_b(z^b, k^b),\ b=1...m,\ \sum_{b=1}^n z^b\leq\sum_{a=1}^m\omega_a^0,\ ((\boldsymbol{x}_a), \boldsymbol{z})\in K$$

Since the set K is compact and convex, the problem $P(k)$ has a solution when the utility function $u_a(\cdot)$ is continuous and concave. We will apply the Kuhn-Tucker theorem (Takayama (1986)).

Theorem (Kuhn-Tucker) Let $f, g_1...g_m$ be real valued concave functions defined on a convex set X in $\mathbb{R}^l$.

Suppose that the Slater's condition holds, namely that

$$\text{there exists an } \boldsymbol{x}_0\in X \text{ such that } g_j(\boldsymbol{x}_0)>0,\ j=1...m.$$

Let $\hat{\boldsymbol{x}}$ be a point which achieves a maximum of $f(\boldsymbol{x})$ on X subject to $g_j(\boldsymbol{x})\geq 0$, $j=1...m$. Then there exists a nonnegative vector $\hat{\boldsymbol{\lambda}}=(\hat{\lambda}_1...\hat{\lambda}_m)\in\mathbb{R}_+^m$ such that

$$f(\boldsymbol{x})+\hat{\boldsymbol{\lambda}}g(\boldsymbol{x})\leq f(\hat{\boldsymbol{x}})+\hat{\boldsymbol{\lambda}}g(\hat{\boldsymbol{x}})\leq f(\hat{\boldsymbol{x}})+\boldsymbol{\lambda}g(\hat{\boldsymbol{x}})$$

for every $\boldsymbol{x}\in X$ and every nonnegative $\boldsymbol{\lambda}\in\mathbb{R}_+^m$.

Then the solution of the problem $P(k)$ is a saddle point of the Lagrangian

$$L_{\alpha,k}((\boldsymbol{x}_a), \boldsymbol{z}, \boldsymbol{p}, w)=\sum_{a=1}^m\alpha^a u_a(\boldsymbol{x}_a)+\sum_{b=1}^n p^b\left(\sum_{b=1}^n f_b(z^b, k_b)-\sum_{a=1}^m x_a^b\right)+\omega\left(\sum_{a=1}^m\omega_a^0-\sum_{b=1}^n z^b\right),$$

where $\boldsymbol{p}=(p^b)\in\mathbb{R}_+^n$ and $w\in\mathbb{R}_+$ are the Lagrangian multipliers.

Let $((\hat{\boldsymbol{x}}_a), \hat{\boldsymbol{z}}, \hat{\boldsymbol{p}}, \hat{w})$ be the saddle point. We can take a constant $b>0$ which atisfies

$$b > sup\left\{\sum_{a=1}^{m}|\, w\omega_a^0 - \boldsymbol{p}\boldsymbol{x}_a \,|\,\middle|\, (\boldsymbol{p}, w)\in\Delta, ((\boldsymbol{x}_a), \boldsymbol{z})\in K\right\}.$$

For given $((\alpha^a), (\boldsymbol{x}_a), \boldsymbol{p}, w)$, define $\hat{\alpha} = (\hat{\alpha}^a)$ by

$$\hat{\alpha}^a = \frac{max\{0, \alpha^a + (1/b)(w\omega_a^0 - \boldsymbol{p}\boldsymbol{x}_a)\}}{\sum_{a=1}^{m} max\{0, \alpha^a + (1/b)(w\omega_a^0 - \boldsymbol{p}\boldsymbol{x}_a)\}},\ a=1...m.$$

Let $Z=\{\boldsymbol{z}\in\mathbb{R}_+^n \,|\, ((\boldsymbol{x}_a), \boldsymbol{z})\in K$ for some $(\boldsymbol{x}_a)\in\mathbb{R}^{mn}\}$. Obviously Z is a compact and convex subset of $\mathbb{R}^n$. Then we can show that the competitive equilibrium is a fixed point of the mapping Φ of $\Delta\times Z\times K\times\Delta$ to itself defined by

$$\Phi : (\alpha, k, (\boldsymbol{x}_a), \boldsymbol{z}, \boldsymbol{p}, w) \mapsto (\hat{\alpha}, \hat{\boldsymbol{z}}, (\hat{\boldsymbol{x}}_a), \hat{\boldsymbol{z}}, \hat{\boldsymbol{p}}, \hat{w}).$$

By the Kakutani's fixed point theorem, there exists a fixed point

$$(\hat{\alpha}, \hat{\boldsymbol{z}}, (\hat{\boldsymbol{x}}_a), \hat{\boldsymbol{z}}, \hat{\boldsymbol{p}}, \hat{w}) \in \Phi\,(\hat{\alpha}, \hat{\boldsymbol{z}}, (\hat{\boldsymbol{x}}_a), \hat{\boldsymbol{z}}, \hat{\boldsymbol{p}}, \hat{w}).$$

We will show that the fixed point is a competitive equilibrium. Since the fixed point is a saddle point of the Lagrangian, we have

$$L_{\hat{\alpha},\hat{z}}((\boldsymbol{x}_a), \boldsymbol{z}, \hat{\boldsymbol{p}}, \hat{w}) \le L_{\hat{\alpha},\hat{z}}((\hat{\boldsymbol{x}}_a), \hat{\boldsymbol{z}}, \hat{\boldsymbol{p}}, \hat{w}) \le L_{\hat{\alpha},\hat{z}}((\hat{\boldsymbol{x}}_a), \hat{\boldsymbol{z}}, \boldsymbol{p}, w)$$

for every $((\boldsymbol{x}_a), \boldsymbol{z})\in K$, and every $\boldsymbol{p}\ge\boldsymbol{0}$ and $w\ge 0$.

It follows from the monotonicity of the utility functions that $\boldsymbol{p}\gg\boldsymbol{0}$ and $w>0$. For if not, we have a contradiction to the first inequality of the saddle point property which implies that the maximality of $L_{\hat{\alpha},\hat{z}}$ with respect to $((\boldsymbol{x}_a), \boldsymbol{z})$. Setting $\boldsymbol{x}_a=\hat{\boldsymbol{x}}_a$, $a=1...m$, and $z_c=\hat{z}_c$, $c\neq b$, it follows from the first inequality and from the homogeneity of the production function with respect to the first variable,

$$\hat{p}^b f_b\left(\hat{z}^b, \sum_{c=1}^{n}\hat{z}^c\right) - \hat{w}\hat{z}^b = 0,\ b=1...n\,.$$

Therefore the profit maximization of the firms in the industry $b=1...n$ is established. Next, the mownotonicity of the utility function implies that $\hat{\alpha}^a>0$, $a=1...m$. For if not, then $\hat{\alpha}^a=0$ for some a. By the definition of $\hat{\alpha}$, for such an a we have $0=max\{0, (1/b)(\hat{w}\omega_a^0 - \hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a)\}$, hence $0<\hat{w}\omega_a^0\le\hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a$. Setting $\boldsymbol{x}_a=\boldsymbol{0}$ and $\boldsymbol{x}_b=\hat{\boldsymbol{x}}_a+\hat{\boldsymbol{x}}_b$ for a consumer b such that $\hat{\alpha}^b>0$, we have a contradiction to the first inequality of the saddle point property. Therefore it follows that

$$\hat{\alpha}^a = \frac{\hat{\alpha}^a + (1/b)(\hat{w}\omega_a^0 - \hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a)}{\sum_{c=1}^{m}\{\hat{\alpha}^c + (1/b)(\hat{w}\omega_c^0 - \hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_c)\}},\ a=1...m,$$

and from this, we obtain

$$\hat{w}\omega_a^0 - \hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a = \hat{\alpha}^a\sum_{c=1}^{m}(\hat{w}\omega_c^0 - \hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_c),\ a=1...m.$$

On the other hand by the second inequality which minimizes the Lagrangian with respect to the multipliers (prices),

$$0=\sum_{a=1}^{m}(\hat{w}\omega_a^0-\hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a)+\sum_{b=1}^{n}(\hat{p}^b f_b(\hat{z}^b,\hat{z}^b)-\hat{w}\hat{z}^b)=\sum_{a=1}^{m}(\hat{w}\omega_a^0-\hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a),$$

hence one obtains

$$\hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a=\hat{w}\omega_a^0,\ a=1...m.$$

Setting $\boldsymbol{x}_a=\boldsymbol{x}$ and $\boldsymbol{x}_b=\hat{\boldsymbol{x}}_b$ for $b\neq a$, we have from the first inequality that

$$\hat{\alpha}u_a(\boldsymbol{x})-\boldsymbol{p}\boldsymbol{x}\leq\hat{\alpha}u_a(\hat{\boldsymbol{x}})-\hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a=\hat{\alpha}^a u_a(\hat{\boldsymbol{x}}_a)-\hat{w}\omega_a^0,$$

or

$$\hat{w}\omega_a^0-\hat{\boldsymbol{p}}\boldsymbol{x}\leq\hat{\alpha}^a(u_a(\hat{\boldsymbol{x}}_a)-u_a(\boldsymbol{x})).$$

Hence $\hat{\boldsymbol{p}}\boldsymbol{x}\leq\hat{w}\omega_a^0$ implies that $u_a(\boldsymbol{x})\leq u_a(\hat{\boldsymbol{x}}_a)$, $a=1...m$. Therefore the utility maximization for each consumer is established.

Finally the market conditions are obvious from the constraint conditions of the problem $P(k)$, indeed they hold with the exact equality by virtue of $\hat{\boldsymbol{p}}\gg\boldsymbol{0}$ and $\hat{w}>0$.

We have obtained the next theorem.

Theorem 3.1 Suppose that the utility functions are continuous, concave and monotone, and each consumer has a strictly positive amount of labor $\omega_a^0>0$ as the initial endowment. Then there exists a competitive equilibrium for the generalized Chipman model.

In the following, we discuss the Pareto optimality of the competitive equilibria. For definiteness, we assume that the utility function is of the Cobb-Douglas form

$$u_a((x_a^b))=\sum_{b=1}^{n}\beta_a^b\log x_a^b,\ \sum_{b=1}^{n}\beta_a^b=1,\ \beta_a^b\geq0,\ a=1...m,\ b=1...n.$$

and we calculate the equilibrium explicitly. The first order conditions for the saddle point property of $L_{\hat{\alpha},\hat{z}}$ are given by

$$\hat{\alpha}^a\frac{\beta_a^b}{\hat{x}_a^b}-\hat{p}^b=0,\ a=1...m,\ b=1...n,$$

$$\hat{p}^b f_b(\hat{z}^b,\hat{z}^b)-\hat{w}\hat{z}^b=0,\ b=1...n.$$

Summing the first mn equations over b and using $\hat{\boldsymbol{p}}\hat{\boldsymbol{x}}_a=\hat{w}\omega_a^0$, we obtain

$$\hat{\alpha}^a=\hat{w}\omega_a^0,\ a=1...m.$$

Summing the same equations over a with help of the above m identities,

$$\hat{w}\sum_{a=1}^{m}\beta_a^b\omega_a^0=\sum_{a=1}^{m}\hat{\alpha}\beta_a^b=\sum_{a=1}^{m}\hat{p}^b\hat{x}_a^b=\hat{p}^b f_b(\hat{z}^b,\hat{z}^b)=\hat{w}\hat{z}^b,$$

hence we have

$$\hat{z}^b=\sum_{a=1}^{m}\beta_a^b\omega_a^0,\ b=1...n.$$

The equilibrium (relative) prices are then determined by

$$\frac{\hat{p}^b}{\hat{w}}=\frac{\hat{z}^b}{f_b(\hat{z}^b,\hat{z}^b)}\ ,\ b=1...n.$$

and the equilibrium consumptions are

$$\hat{x}_a^b=\left(\frac{\hat{w}}{\hat{p}^b}\right)\beta_a^b\omega_a^0=\left(\frac{\beta_a^b\omega_a^0}{\sum_{a=1}^m\beta_a^b\omega_a^0}\right)f_b(\hat{z}^b,\hat{z}^b)\ ,\ a=1...m,\ b=1...n.$$

We now turn to the Pareto optimal allocations. Let $g_b(z^b)=f_b(z^b, z^b)$ and consider the social optimization problem

P : Given $\alpha=(\alpha^a)\in\mathbb{R}^m_+$,

maximize $\sum_{a=1}^m\alpha^a\sum_{b=1}^n\beta_a^b\log x_a^b$ subject to

$$\sum_{a=1}^m x_a^b\leq g_b(z^b)\ ,\ b=1...m,\ \sum_{b=1}^n z^b\leq\sum_{a=1}^m\omega_a^0\ ,\ ((\mathbf{x}_a),\ \mathbf{z})\in K.$$

Note that the problem P does not depend on k any more. We want to compare the solution $(\tilde{\mathbf{x}}_a)$, $(\tilde{z}_b))$ of P with the equilibrium allocation $((\hat{\mathbf{x}}_a),\ (\hat{z}_b))$. The problem is what a social weight $\alpha=(\alpha^a)$ should we use for the social welfare maximization problem P? Fortunately, we have already obtained the answer, namely that we should use $\alpha^a=\hat{\alpha}^a=\hat{w}\omega_a^0$, $a=1...m$. Then the first order conditions for theproblem P are written as

$$\hat{\alpha}^a\frac{\beta_a^b}{\tilde{x}_a^b}-\lambda^b=0\ ,\ a=1...m,\ b=1...n,$$

$$\lambda^b g_b'(\tilde{z}^b)-\mu=0,\ b=1...n,$$

where λ^b and μ are the Lagrangian multipliers.

Summing the first mn equations over a, we obtain

$$\lambda^b=\frac{\sum_{a=1}^m\hat{\alpha}^a\beta_a^b}{\sum_{a=1}^m\tilde{x}_a^b}=\frac{\sum_{a=1}^m\hat{\alpha}^a\beta_a^b}{g_b(\tilde{z}^b)}\ ,\ b=1...n.$$

Substituting these equalities to the second group of n equations of the first order conditions, we have

$$\mu=\left(\sum_{a=1}^m\hat{\alpha}^a\beta_a^b\right)(g_b'(\tilde{z}^b)/g_b(\tilde{z}^b))=\left(\hat{w}\ \sum_{a=1}^m\beta_a^b\omega_a^0\right)(g_b'(\tilde{z}^b)/g_b(\tilde{z}^b))\ ,\ b=1...n.$$

Multiplying these equations by $\tilde{z}^b$ and summing over b with the help of $\sum_{b=1}^n\tilde{z}_b=\sum_{a=1}^m\omega_a^0$, we obtain

$$\mu=\hat{w}\sum_{b=1}^n\left(\frac{\sum_{a=1}^m\beta_a^b\omega_a^0}{\sum_{a=1}^m\omega_a^0}\right)\left(\frac{g_b'(\tilde{z}^b)\,\tilde{z}^b}{g_b(\tilde{z}^b)}\right)=\hat{w}\sum_{b=1}^n\left(\frac{\sum_{a=1}^m\beta_a^b\omega_a^0}{\sum_{a=1}^m\omega_a^0}\right)\varepsilon_b(\tilde{z}^b)\ ,$$

where $\varepsilon_b(\tilde{z}^b)=g_b'(\tilde{z}^b)\tilde{z}^b/g_b(\tilde{z}^b)$ is the elasticity of the production for the firm b. From this and the previous n equations we obtain

$$\tilde{z}^b=\left(\frac{\varepsilon_b(\tilde{z}^b)}{\sum_{b=1}^{n}\left(\sum_{a=1}^{m}\beta_a^b\omega_a^0/\ \sum_{a=1}^{m}\omega_a^0\right)\varepsilon_b(\tilde{z}^b)}\right)\sum_{a=1}^{m}\beta_a^b\omega_a^0,\ b=1...n,$$

and

$$\tilde{x}_a^b=\frac{\hat{\alpha}^a\beta_a^b}{\lambda^b}=\left(\frac{\beta_a^b\omega_a^0}{\sum_{a=1}^{m}\beta_a^b\omega_a^0}\right)g_b(\tilde{z}^b)\ ,\ a=1...m,\ b=1...n.$$

In view of $\hat{\boldsymbol{x}}_a$, $\hat{\boldsymbol{z}}$, $\hat{\boldsymbol{x}}_a$ and $\tilde{\boldsymbol{z}}$, we see that

$$\tilde{x}_a^b\geq\hat{x}_a^b \text{ if and only if } \tilde{z}^b\geq\tilde{z}^b,\ a=1...m,\ b=1...n.$$

This is equivalent to

$$\varepsilon_b(\tilde{z}^b)\geq\sum_{b=1}^{n}\chi_b\varepsilon_b(\tilde{z}^b)\ ,\ b=1...n,$$

where $\chi_b=\sum_{a=1}^{m}\beta_a^b\omega_a^0/\sum_{a=1}^{m}\omega_a^0$, $b=1...n$.

Therefore we have obtained the next fundamental result on the optimality of the competitive equilibria.

Theorem 3.2 Under the above assumptions, optimal output of the b-th producer is greater than, equal to, or less than competitive output according as the elasticity of the industry (at optimum) is greater than, equal to, or less than the weighted average of elasticities of all industries. If all industries' elasticities are equal to the weighted average, the competitive equilibrium is Pareto optimal.

In the following, we assume that the objective production function is of the constant elasticity form

$$g_b(z)=\kappa_b z^{\varepsilon b},\ b=1...n$$

and we will discuss the tax policies to implement the optimal competitive equilibrium. We recall the consumers and the producers optimal conditions,

$$p^b x_a^b=\hat{w}\beta_a^b\omega_a^0,\ a=1...m,$$

where we set $\alpha^a=\hat{\alpha}^a=\hat{w}\omega_a^0$, and

$$q^b g_b(z^b)=\hat{w}z^b,\ b=1...n.$$

First we consider the case of per-unit exercise taxes,

$$p^b-q^b=t^b,\ b=1...n.$$

Then we have

$$t^b g_b(z^b)=p^b\sum_{a=1}^{m}x_a^b-q^b g(z^b)=\hat{w}\left(\sum_{a=1}^{m}\beta_a^b\omega_a^0-\ z^b\right),\ b=1...n.$$

Hence $\sum_{b=1}^{n}t^b g_b(\hat{z}^b)=0$, or the total subsidies paid is equal to the total taxes collected. Setting $z^b=\tilde{z}^b$, the optimal tax rates are calculated as

$$\tilde{t}^b=\frac{\hat{w}}{\kappa_b}\left(1-\frac{\varepsilon_b}{\sum_{b=1}^{n}\chi_b\varepsilon_b}\right)^{-\varepsilon b}\left(\sum_{a=1}^{m}\beta_a^b\omega_a^0\right)^{1-\varepsilon b},\ b=1...n.$$

Therefore $\tilde{t}^b \geq 0$ if and only if $\sum_{b=1}^{n} \chi_b \varepsilon_b \geq \varepsilon_b$. When $\sum_{b=1}^{n} \chi_b \varepsilon_b = 1$, this reduces to Marshall's rule (*Principles*, 8-th ed., pp. 388-390.) followed by Pigou (*Wealth and Welfare*, p. 178.) which asserts that one should levy taxes on the industry which exhibits the decreasing returns to scale ($\varepsilon_b<1$) and should subsidize the increasing returns to scale industries.

Next we consider the case of ad valorem taxes,

$$p^b - q^b = \tau^b p^b,\ b=1...n.$$

In this case, the optimal tax rates are calculated as

$$\tilde{\tau}^b = 1 - \frac{q^b g_b(\tilde{z}^b)}{p^b \sum_{a=1}^{m} \tilde{x}_a^b} = 1 - \frac{\tilde{z}^b}{\sum_{a=1}^{m} \beta_a^b \omega_a^0} = 1 - \frac{\varepsilon_b}{\sum_{b=1}^{n} \chi_b \varepsilon_b},\ b=1...n.$$

Consider the (objective) marginal cost

$$\delta_b = \frac{d}{dy^b} \hat{w} z^b = \frac{\hat{w}}{g_b'(z^b)},\ b=1...n,$$

and the market value of the objective marginal product

$$\rho_b = p^b g_b'(z^b),\ b=1...n.$$

Then it follows that

$$\frac{p^b}{\delta_b} = \frac{\rho_b}{\hat{w}} = \frac{p^b g_b'(z^b) z^b}{\hat{w} z^b} = \varepsilon_b \frac{p^b g_b(z^b)}{\hat{w} z^b} = \varepsilon_b \frac{p^b \sum_{a=1}^{m} x_a^b}{\hat{w} z^b} = \varepsilon_b \frac{\sum_{a=1}^{m} \beta_a^b \omega_a^0}{z^b},\ b=1...m.$$

Hence in the competitive equilibrium, setting $z^b = \hat{z}^b = \sum_{a=1}^{m} \beta_a^b \omega_a^0$, we obtain

$$\frac{\hat{p}^b}{\delta_b} = \frac{\rho_b}{\hat{w}} = \varepsilon_b,\ b=1...n.$$

whereas, the optimal competitive equilibrium $z^b = \tilde{z}^b = \left(\varepsilon_b / \sum_{b=1}^{n} \chi_b \varepsilon_b\right) \sum_{a=1}^{m} \beta_a^b \omega_a^0$ requires

$$\frac{\tilde{p}^b}{\delta_b} = \frac{\tilde{\rho}_b}{\hat{w}} = \sum_{b=1}^{n} \chi_b \varepsilon_b,\ b=1...n.$$

This is the proportionality rule developed by Pigou and Kahn; for all commodities, price must be proportional to the marginal costs, and the wage rate to the value of the marginal product, the factor of proportionality being the average of the production elasticities, $\sum_{b=1}^{n} \chi_b \varepsilon_b$. A.C. Pigou wrote in his *The economics of welfare* 4-th edition, p. 225,

> "When it was urged above that in certain industries a wrong amount of resources is being invested because the value of the marginal social net product there differs from the value of the marginal private net product, it was tacitly assumed that in the main body of industries these two values are equal... If in all industries that value of marginal social and marginal private net product differed to exactly the same extent, the *optimum* distribution of resources would always be attained and there would be ... no case for financial nterference."

If the average value $\sum_{b=1}^{n} \chi_b \varepsilon_b$ is equal to 1, we have the special case in which price should be set equal to the marginal costs, and the wage rate to the marginal product. This is nothing but the marginal cost pricing (MCP) rule.

4 Notes

Section 2: As stated in Section 1, the concept of the external increasing returns was originally due to Edgeworth. It was formulated in the modern theoretical framework by Chipman (1970) and Romer (1986). Extending their results to the case of several consumers, the general equilibrium analysis was started by Suzuki (1992). The expositions in this section are based on Suzuki (1996, 2008).

Section 3: The expositions of this section are essentially due to Chipman (1970) and Suzuki (2008). In *Principles*, p. 389, A. Marshall wrote

> "... By similar reasoning, it may be shown that a tax on a commodity which obeys the law of increasing return is more injurious to the consumer than if levied on one which obeys the law of constant return. For it lessens the demand and therefore the output. It thus probably increases the expenses of manufacture somewhat: sends up the price by more than the amount of the tax; and finally diminishes consumers' surplus by much more than the total payments which it brings in to the exchequer. On the other hand, a bounty on such a commodity causes so great a fall in its price to the consumer, that the consequent increase of consumers' surplus may exceed the total payments made by the State to the producers; and certainly will do so in case the law of increasing return acts at all sharply."

On the light of the analysis of this section, the theoretical background of the so called "Pigou tax" originally due to Marshall cited above seems to become clear.

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