『経済研究』（明治学院大学）第 151 号　2016 年

Inter-Industrial Integration: M&A with Related Goods in Demand*

Jinryo TAKASAKI
Department of Economics
Meiji Gakuin University

1. Introduction

It has been known that if two firms whose products are perfectly complementary each other merge or collude, then the consumer prices of the complements lower (Cournot [2], Tirole [8], Economedes and Salop [3]). However we find very few examples of the perfect complements in the real consumer's markets. Takasaki [7] showed that the almost same result follows when the products are 'imperfect' complements under moderate conditions.

Takasaki [7] also includes a generalization of perfectly substitute goods case, i.e. we show that the merger in the case of 'imperfect' substitutes tend to make consumer prices (quantities of the goods) higher (decrease).

After Takasaki [7], many researches with the same results appeared. However they are based on the assumption of simple linear demand curves. Our general models also show the limits of their analysis.

2. Basic Definitions and Assumptions

We assume that any function used in this paper is twice continuously differentiable. Other basic and technical assumptions are summarized by Assumption 1 and 2.

---

* This paper is a revised version of Takasaki [7].

(Definition 1)　There are n goods. Let x_i and p_i denote the quantity and the price of good i ($i=1, 2, ..., n$). The market demand functions of these goods, $D^i : \mathbf{R}^n_{++} \rightarrow \mathbf{R}_+$, are

$$x_i = D^i(p) \qquad i=1, 2, \ldots, n \tag{1}$$

where $p=(p_1, p_2, \ldots, p_n)$. Equation(1) is also expressed by $x=D(p)$ using the notations $x=(x_1, x_2, \ldots, x_n)^T$ and $D(p)=(D^1(p), D^2(p), \ldots, D^n(p))^T$ where T means transposition of vector.

(Definition 2)　Let O be an open set in $\mathbf{R}^n_{++}$ and let D^i_j denote $\partial x_i/\partial p_j$. We call goods i and j ($i \neq j$) 'gross substitutes in O' if

$$D^i_j > 0, D^j_i > 0 \text{ for all } p \in \mathrm{O} \tag{2},$$

and we call them 'gross complements in O' if

$$D^i_j < 0, D_j^{\,i} < 0 \text{ for all } p \in \mathrm{O} \tag{3}.$$

Hereafter, we call 'gross substitutes' and 'gross complements' simply 'substitutes' and 'complements' respectively.

(Assumption 1)　$D(p)$ has its inverse $p(x) \equiv (p_1(x), p_2(x), \ldots, p_n(x))^T$ and

$$D^i_i < 0, \qquad (-1)^m \begin{vmatrix} D^1_1 & \ldots\ldots & D^1_m \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ D^m_1 & \ldots\ldots & D^m_m \end{vmatrix} > 0 \qquad i=1, 2, \ldots, n \qquad m \leq n \tag{4}.$$

Equation (4) is interpreted as Hicksian stability condition when the supplies are fixed.

(Definition 3)　The total cost function of good i is denoted by

$$C^i = C^i(x_i) \qquad i=1, 2, \ldots, n \tag{5}$$

and its marginal cost function is denoted by

$$MC^i = dC^i/dx_i \qquad i=1, 2, \ldots, n \tag{6}.$$

In order to extract pure effect of complementarity or substitution in consumption, we exclude such technological properties of multi-product cost function as 'economies of scope' or 'cost complementarity'.

(Definition 4)　The profit function of a single-product firm which produces good i is

$$\pi^i = \pi^i(x) = x_i p_i(x) - C^i(x_i) \tag{7}$$

or alternatively

$$\pi^i = \pi^i(D(p)) = D^i(p)p_i - C^i(D^i(p)) \tag{8},$$

and the marginal profit function is denoted by

$$\phi^i(x) \equiv \partial \pi^i/\partial x_i \tag{9}$$

or

$$\psi^i(p) \equiv \partial\pi^i/\partial p_i \qquad (10).$$

The profit function of a multi-product firm is

$$\sum_{i \in S} \pi^i \qquad (11)$$

where S is the set of the goods produced by the firm.

(Assumption 2) Each profit maximization problem (not only for a single-product firm but also for a multi-product firm) has the unique and positive solution.

(Definition 5) Define $\phi_j^i \equiv \partial\phi^i/\partial x_j$. Quantities x_i and x_j are called 'strategic substitutes' when the following inequality holds:

$$\phi_j^i < 0 \qquad \phi_i^j < 0 \qquad i \neq j \qquad (12)$$

and called 'strategic complements' when the following holds: (See [1] 1985 in the references.)

$$\phi_j^i > 0 \qquad \phi_i^j > 0 \qquad i \neq j \qquad (13).$$

(Definition 6) Define $\psi_j^i \equiv \partial\psi^i/\partial p_j$. Prices p_i and p_j are called 'strategic substitutes' when the following inequalities hold:

$$\psi_j^i < 0 \qquad \psi_i^j < 0 \qquad i \neq j \qquad (14)$$

and called 'strategic complement' when the following inequalities hold: (See also [1] 1985.)

$$\psi_j^i > 0 \qquad \psi_i^j > 0 \qquad i \neq j \qquad (15).$$

(Definition 7) An open interval $\{u : a < u < b\} \subset \mathbf{R}_+^n$ with $a, b \in \mathbf{R}_+^n$ is called a 'relevant region of y and z' if y and z belong to the interval.

The definitions and assumptions in this section are commonly used below. And we often denote the solutions for quantities (prices) before and after merger by x° and x^* (p° and p^*) respectively.

3. Quantity Competition and Merger

3-1 Basic Analysis

In this basic analysis we investigate the effect of merger between 2 firms on quantities under some additional assumptions. We consider alternative definitions of 'substitutes' and 'complements'.

(Definition 8) Let O′ be an open set in $\mathbf{R}_{++}^n$. We call goods i and j ($i \neq j$) 'substitutes in O′' if

$$\partial p_i / \partial x_j < 0 \qquad \partial p_j / \partial x_i < 0 \qquad \text{for all } x \in O' \tag{16}$$,

and we call them 'complements in O′' if

$$\partial p_i / \partial x_j > 0 \qquad \partial p_j / \partial x_i > 0 \qquad \text{for all } x \in O' \tag{17}$$.

The meaning of Definition 8 is as follows: When x_j increases, consumers try to decrease (increase) x_i if these goods are substitute (complementary). In order to keep x_i constant (a meaning of partial derivatives), p_i must lower (rise). We show that Definition 8 is equivalent to Definition 2 under our assumptions when $n=2$. Substitute $p(x)$ into $D(p)$, and differentiate the identity $x \equiv D(p(x))$ by x. Then we have $I = \partial x/\partial p \cdot \partial p/\partial x$ (I is the identity matrix) or alternatively $[\partial x/\partial p]^{-1} = \partial p/\partial x$. When $n=2$, the latter equation is expressed more precisely as follows:

$$\begin{bmatrix} D_2^2 & -D_2^1 \\ -D_1^2 & D_1^1 \end{bmatrix} = \begin{bmatrix} \partial p_1/\partial x_1 & \partial p_1/\partial x_2 \\ \partial p_2/\partial x_1 & \partial p_2/\partial x_2 \end{bmatrix} \begin{vmatrix} D_1^1 & D_2^1 \\ D_1^2 & D_2^2 \end{vmatrix} \tag{18}$$.

The determinant of the right-hand side of (18) is positive by Assumption 1 (equation (4)). Therefore Definition 2 and 8 are equivalent.

We assume that firm i produces only good i ($i=1, 2$) and compete in quantity, then we investigate the effect of merger or collusion. Before merger or collusion, each firm maximizes its own profit π^i. After merger or collusion, they maximize their joint profit $\pi^1 + \pi^2$. The first order condition for the former case is

$$\phi^i(x_1, x_2) = \frac{\partial p_i}{\partial x_i} x_i + p_i - MC^i = 0 \qquad \mathrm{i} = 1, 2 \tag{19}$$

and that for the latter case is

$$\phi^i(x_1, x_2) = -\frac{\partial p_j}{\partial x_i} x_j \qquad i, j = 1, 2 \qquad i \neq j \tag{20}$$.

We denote the solutions of (19) and (20) by (x_1^o, x_2^o) and (x_1^*, x_2^*) respectively. We have assumed the existence of the unique positive solution (the second order condition) for both cases (Assumption 2).

(Assumption 3) The following inequalities hold in a relevant region of (x_1^o, x_2^o) and (x_1^*, x_2^*).

$$\phi_i^i < 0, \qquad \begin{vmatrix} \phi_i^i & \phi_j^i \\ \phi_i^j & \phi_j^j \end{vmatrix} > 0 \qquad i, j = 1, 2 \qquad i \neq j \tag{21}$$.

This assumption corresponds to the stability condition of Cournot (i.e. quantity competition) model.

Let us define function f as follows:

$$f(x_1, x_2) \equiv (-\phi^1(x_1, x_2), -\phi^2(x_1, x_2))^T \tag{22}$$.

Jacobian of f and its principal minors (i.e. $-\phi_1^1$ and $-\phi_2^2$) are all positive by Assumption 3. When two goods are complements in the relevant region, the right-hand side of equation (20) is negative. According to equations of (19) and (20), it follows that

$$f(x_1^o, x_2^o) < f(x_1^*, x_2^*) \tag{23}$$

By theorem 3 of Gale and Nikaido [4] 1965, inequality (23) has no solution in any region with $(x_1^o, x_2^o) \geq (x_1^*, x_2^*)$. Therefore we arrive at the following proposition.

(Proposition 1) In the case of quantity competition, merger or collusion between two firms who produce complementary goods increases the quantity of at least one of the two goods.

When these goods are substitutes, the right-hand side of equation (20) is positive by the definition. We can obtain another proposition by similar discussion as previous.

(Proposition 2) In the case of quantity competition, merger or collusion between two firms who produce substitute goods decreases the quantity of at least one of the two goods.

When goods are complements and $x_i \partial^2 p_i / \partial x_i \partial x_j$ is positive or negligible, inequality (13) in Definition 5 holds. Therefore 'strategic complements' case is highly plausible.

(Proposition 3) If two firms compete in quantity and the quantities are strategic complements in a relevant region, merger or collusion between two firms who produce complementary goods increases the quantities of both goods.

The proof is as follows: If $x_i^* \leq x_i^o$ for some i, then $x_j^* > x_j^o$ for $j \neq i$ by Proposition 1. The function ϕ^i is a decreasing function of x_i in a relevant region by Assumption 3 and an increasing function of x_j because the quantities are strategic complements. Therefore it must be that $\phi^i(x_1^o, x_2^o) < \phi^i(x_1^*, x_2^*)$. This result contradicts equations (19) and (20) with our definition of complementary goods.

Furthermore, recalling the same argument that proved Proposition 1 and 2, we obtain the following proposition.

(Proposition 4) If two firms whose products are complementary compete in quantity and the quantities are strategic complements, the merger or collusion between the firms lowers the price of at least one of the two goods.

This proposition holds because the Jacobian of $-D(p)$ and its principal minors are positive by

Assumption 1, and both quantities increase when the firms merge by Proposition 3 (again we use theorem 3 of Gale and Nikaido [4] 1965).

We show an example in which both quantities increase (decrease) and both prices lower (rise) by merger in complementary (substitute) goods case.

Demand functions:

$$\left.\begin{aligned} x_1 &= ap_1 + bp_2 + c \\ x_2 &= ap_2 + bp_1 + c \end{aligned}\right\}$$

$$a < 0,\ c > 0,\ a^2 - b^2 > 0 \tag{24}$$

$b > 0$ if goods are substitutes.

$b < 0$ if goods are complements.

Cost functions: $$C^i = mx_i, \qquad m > 0, \qquad i = 1, 2 \tag{25}$$

Here c is sufficiently large so that $am + bm + c > 0$ (This assumption is quite reasonable because $am + bm + c$ is the quantity demanded by consumers when both firms adopt marginal cost pricing).

The solutions in this example are as follows:

$$x_i^o = \frac{(a-b)(am+bm+c)}{2a-b} \qquad i = 1, 2 \tag{26}$$

$$x_i^* = \frac{am+bm+c}{2} \qquad i = 1, 2 \tag{27}$$

$$p_i(x_1^o, x_2^o) = \frac{(a^2-b^2)m - ac}{(2a-b)(a+b)} \qquad i = 1, 2 \tag{28}$$

$$p_i(x_1^*, x_2^*) = \frac{(a+b)m - c}{2(a+b)} \qquad i = 1, 2 \tag{29}$$

It is easily seen that $x_i^* < x_i^o$ ($x_i^* > x_i^o$) and $p_i(x_1^*, x_2^*) > p_i(x_1^o, x_2^o)$ ($p_i(x_1^*, x_2^*) < p_i(x_1^o, x_2^o)$) in substitute (complementary) goods case.

3-2 Further Analysis

Let us assume that there are n goods. We denote the solution before merger by x^o and the solution after merger by x^*.

(Definition 9) Let us consider $\{M_1, M_2, \ldots, M_m\}$ such that $\bigcup_{s=1}^{m} M_s = \{1, 2, \ldots, n\}$. Interpret '$i \in M_s$' as 'firm s produces good i'. We call $\{M_1, M_2, \ldots, M_m\}$ an 'industry configuration' where m is the number of firms.

(Assumption 4) The inverse demand function is (approximately) expressed in linear form by

$p = Ax + b$ in a relevant region of x^{o} and x^{*}. Here $b = (b_1, b_2, \dots, b_n)^T$ and

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \tag{30}$$

with the condition that $a_{ii} < 0$ for all i.

(Assumption 5) The marginal cost of each good is constant and non-negative in a relevant region of x^{o} and x^{*}. We denote the constant marginal cost of good i by mc_i.

(Proposition 5) Suppose there are n firms, firm i produces only good i ($i = 1,2, \dots, n$), they compete in quantity and any two goods are complementary according to both Definition 2 and 8. If some of n firms merge and consequently form an industry configuration $\{M_1, M_2, \dots, M_m\}$, then $x^{*} \geq x^{\mathrm{o}}$ and $x^{*} \neq x^{\mathrm{o}}$ under Assumption 1, 4 and 5.

This proposition is proved as follows: the first order condition of profit maximization before merger is

$$p_i^{\mathrm{o}} + a_{ii} x_i^{\mathrm{o}} = mc_i .$$

After merger, it is

$$p_i^{*} + a_{ii} x_i^{*} = mc_i \qquad \text{for non-merged firms,}$$

and

$$p_i^{*} + a_{ii} x_i^{*} = mc_i - \sum_{j \in Ms}^{j \neq i} a_{ji} x_j^{*}, \ 1 \leq s \leq m \text{ for merged firms.}$$

Since goods i and j are complements ($i \neq j$), $a_{ij} > 0$. Define

$$B \equiv \begin{bmatrix} 2a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & 2a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & 2a_{nn} \end{bmatrix} \tag{31}.$$

From the above first order conditions it follows that $B(x^{*} - x^{\mathrm{o}}) \leq 0$ or $-B(x^{*} - x^{\mathrm{o}}) \geq 0$. Let A^{-1} be the inverse of A. Since $-A^{-1}$ is a positive matrix by Definition 2, $-A$ satisfies Hawkins-Simon's condition. This means that $-B$ also satisfies the condition. Therefore equation $-B(x^{*} - x^{\mathrm{o}}) \geq 0$ has non-negative solution i.e. $x^{*} \geq x^{\mathrm{o}}$. It is easily seen that $x^{*} \neq x^{\mathrm{o}}$.

Alternatively let us consider the following assumption.

(Assumption 6) For each i, $\varepsilon_i \equiv \dfrac{\partial p_i}{\partial x_i} \dfrac{x_i}{p_i}$, 'demand elasticity of price', is constant in a relevant region.

(Proposition 6) Suppose there are n firms, firm i produces only good i ($i = 1, 2, \ldots, n$) and they compete in quantity. If some of n firms merge, consequently form an industry configuration $\{M_1, M_2, \ldots, M_m\}$ and, for some s with $1 \leq s \leq m$, M_s has more than one element and any two goods in M_s are complementary (substitute), then $p_i^* < p_i^o$ ($p_i^* > p_i^o$) for $i \in M_s$ under Assumption 1, 5 and 6.

Consider M_s which has more than one element. For $i \in M_s$, the first order condition of profit maximization is

$$\frac{\partial p_i}{\partial x_i} x_i^o + p_i^o - mc_i = (\varepsilon_i + 1)\, p_i^o - mc_i = 0 \tag{32}$$

before merger. From this equation we know that $\varepsilon_i \geq -1$. The first order condition of profit maximization after merger is

$$\frac{\partial p_i}{\partial x_i} x_i^* + p_i^* - mc_i = (\varepsilon_i + 1)\, p_i^* - mc_i = -\sum_{j \in M_s}^{j \neq i} \frac{\partial p_j}{\partial x_i} x_j^* \tag{33}.$$

Right hand side of the above equation is negative (positive) in the complementary (substitute) goods case. Since ε_i and mc_i are constant, it must be that $mc_i > 0$ and $\varepsilon_i > -1$ in a relevant region. It is easily seen that $p_i^* < p_i^o$ ($p_i^* > p_i^o$) for $i \in M_s$ in complements (substitutes) case. This proves Proposition 6.

4. Price Competition and Merger

4-1 Basic Analysis

Let us consider the following situation. There are 2 firms. Firm 1 produces good 1 and firm 2 produces good 2. Their strategies are prices. The first order condition for profit maximization of each firm is

$$\psi^i(p_1, p_2) = x_i + (p_i - MC^i) D_i^i = 0 \qquad i = 1, 2 \tag{34}.$$

When they merge or collude, the first order condition of joint profit maximization is

$$\left.\begin{aligned} x_1 + D_1^1(p_1 - MC^1) + D_1^2(p_2 - MC^2) = 0 \\ x_2 + D_2^1(p_1 - MC^1) + D_2^2(p_2 - MC^2) = 0 \end{aligned}\right\} \tag{35}.$$

Equation (35) proves that, in complementary goods case, it is impossible that all prices are less than or equal to their respective marginal costs. In substitutes case, each price can not be less than the marginal cost because the coefficients matrix of (35) satisfies Hawkins-Simon's condition by Assumption 1.

(Proposition 7) When a firm produces two complementary goods, at least one price must exceed the marginal cost.

Many cases wherein $p_i \leq MC^i$ for some i correspond to two-part tariffs; this was first investigated by W. Oi [6] 1971. These cases are caused by extraordinarily asymmetric demand structure. We may exclude such asymmetric cases from our complementary goods analysis. Combining with Proposition 7, however, the following assumption supports one more general proposition.

(Assumption 7) Let (p_1^o, p_2^o) denote the solution of equation (34) and (p_1^*, p_2^*) the solution of equation (35). Prices are strategic substitute (see Definition 6) in a relevant region of (p_1^o, p_2^o) and (p_1^*, p_2^*) when the goods are complements in the region.

By Proposition 7, it must be that, in complementary goods case,

$$\psi^i(p_1^*, p_2^*) > \psi^i(p_1^o, p_2^o) = 0 \tag{36}$$

for some i. ψ^i is decreasing function of p_i in a relevant region. By Assumption 7, it is impossible that $(p_1^*, p_2^*) \geq (p_1^o, p_2^o)$.

(Proposition 8) When two single-product firms' products are complementary each other and they compete in price, the merger or collusion between these two firms lowers at least one price under Assumption 7.

When we assume the stability condition of price competition i.e. following Assumption 8, the corresponding result to Proposition 8 for substitute goods case is obtained.

(Assumption 8)

$$\psi_i^i < 0, \qquad \begin{vmatrix} \psi_i^i & \psi_j^i \\ \psi_i^j & \psi_j^j \end{vmatrix} > 0, \qquad\qquad i, j = 1, 2 \qquad i \neq j$$

(Proposition 9) When two single-product firms' products are substitute each other and they compete in price, the merger or collusion between these two firms raise at least one price under Assumption 8.

This is because Jacobian of $-\psi^i$ and its principal minors are positive by Assumption 8 and we have the same argument as in the previous section using theorem 3 of Gale and Nikaido [4] 1965.

The example in the previous section (equations (24) and (25)) has symmetric demand structure and the following solutions in price competition case:

$$p_i^{\circ} = \frac{am - c}{2a + b} \qquad i = 1, 2 \tag{37}$$

$$p_i^{*} = \frac{(a + b)m - c}{2(a + b)} \qquad i = 1, 2 \tag{38}$$

$$D^i(p_1^{\circ}, p_2^{\circ}) = \frac{a(am + bm + c)}{2a + b} \qquad i = 1, 2 \tag{39}$$

$$D^i(p_1^{*}, p_2^{*}) = \frac{(a + b)m + c}{2} \qquad i = 1, 2 \tag{40}$$

It is easily seen that $p_i^{*} < p_i^{\circ}$ and $D^i(p_1^{*}, p_2^{*}) > D^i(p_1^{\circ}, p_2^{\circ})$ in complementary goods case and the inequalities are inverted in substitute goods case.

4-2 Further Analysis

As we saw in the basic analysis, the case that the marginal cost exceeds the price of a good of multi-product firm is caused by such an extremely asymmetric structure in demand functions as makes two-part tariffs profitable. Here we exclude the extreme case by the following assumption.

(Assumption 9) At joint profit maximization, each price exceeds its corresponding marginal cost.

(Assumption 10) For each i, price elasticity of demand i.e. $\eta_i = \frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i}$ is constant and negative in a relevant region.

(Proposition 10) Suppose that there are n single-product firms, firm i produces good i ($i = 1, 2, \cdots, n$) and they compete in price, then some of them merge and form an industry configuration $\{M_1, M_2, \ldots, M_m\}$. If, for some s with $1 \leq s \leq m$, M_s has more than one element and any two goods in M_s are complementary (substitute), then the prices of the goods in M_s lower (rise) under Assumption 2, 5, 9 and 10.

This proposition is proved as follows. Before merger, the first order condition for profit maximization is

$$x_i^{\circ} + \frac{\partial x_i}{\partial p_i}(p_i^{\circ} - mc_i) = 0 \tag{41}$$

After merger, the first order condition for profit maximization of merged firm is

$$x_i{}^* + \frac{\partial x_i}{\partial p_i}(p_i{}^* - mc_i) = -\sum_{j \in Ms}^{j \neq i} \frac{\partial x_j}{\partial p_i}(p_j - mc_j) \text{ for } i, j \in M_s \tag{42}$$

This is positive in complements case and negative in substitutes case by Assumption 9. By Assumptions 5 and 10, mc_i and η_i are constant. By Assumption 2, x_i^{o} and x_i^* are positive. Therefore we can divide equation (41) by x_i^{o} and (42) by x_i^*. In complementary goods case, we have the following inequality.

$$\eta_i(1 - \frac{mc_i}{p_i^*}) > \eta_i(1 - \frac{mc_i}{p_i^{o}}) \tag{43}$$

The inequality in (43) is inverted in substitute goods case. Therefore we obtain the result $p_i^* < p_i^{o}$ $(p_i^* > p_i^{o})$ for complements (substitutes) case.

(Proposition 11) Consider the same situation as in Proposition 5 but let us assume that firms compete in price and the demand functions are expressed by $x = Cp + d$ in a relevant region where

$$x = Cp + d,\ C = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \cdots & \cdots & \cdots \\ c_{n1} & \cdots & c_{nn} \end{bmatrix} \text{ and } \quad d = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}.$$

If any two goods are substitute according to both Definition 2 and 8, then $p^* \geq p^{o}$ and $p^* \neq p^{o}$ under Assumption 1, 5 and 9.

This is proven quite similarly as we have proved Proposition 5. Define D as follows.

$$D \equiv \begin{bmatrix} 2c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & 2c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ c_{n1} & c_{n2} & \cdots & 2c_{nn} \end{bmatrix}$$

Since $c_{ij} > 0$ for $i \neq j$, $-C^{-1}$ is a positive matrix by Assumption 1 and Definition 8, and $-C$ satisfies Hawkins-Simon's condition. Therefore $-D$ also satisfies the condition. We obtain $-D\ (p^* - p^{o}) \geq 0$ similarly as in the proof of Proposition 5. Therefore $p^* \geq p^{o}$ and $p^* \neq p^{o}$.

5. Concluding Remark

We generalized classical results that consumer prices lower (rise) when firms with perfectly complementary (substitute) goods merge or collude. That is, if firms with imperfectly complementary (substitute) goods merge or collude, consumer prices still tend to lower (rise). Joint profit maximization generally increase producer's surplus and lower prices increase consumer's surplus. Therefore inter-industrial integration with complementary goods increases social welfare. We also showed the limits of the present analysis by using general non-linear demand functions. So far, some of our assumptions appear still confining. We hope further investigation improves our analysis.

[References]

[1] Bulow, J., Geamakoplos, and P. Klemperer, 1985, Multimarket Oligopoly: Strategic Substitutes and Complements, Journal of Political Economy 93, 488–511.

[2] Cournot, A., 1927, Researches into the Mathematical Principles of the Theory of Wealth (original published in French, 1838, translated by Nathaniel Bacon, New York: Macmillan).

[3] Economides, N., and Salop, S., 1992, Competition and Integration among Complements and Network Structure, Journal of Industrial Economics 40, 105–123.

[4] Gale, D. and Nikaido, H., 1965, Jacobian Matrices and Global Univalence of Mappings, Mathematishe Annalen 159, 81–93.

[5] Hawkins, D., and Simon, H.A., 1949, Some Conditions of Macroeconomic Stability, Econometrica 17.

[6] Oi, W.Y., 1971, A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse Monopoly, Quarterly Journal of Economics 85, 77–90.

[7] Takasaki, J., 1983, Inter-industrial Merger: The Case of Complementary Goods, Keizai Ronsou, Kyoto University, 132 (5・6) (Written in Japanese Language).

[8] Tirole, J., 1988, The Theory of Industrial Organization (The MIT Press).